Automatic Verification of non-silent Population Protocols

Master's Thesis

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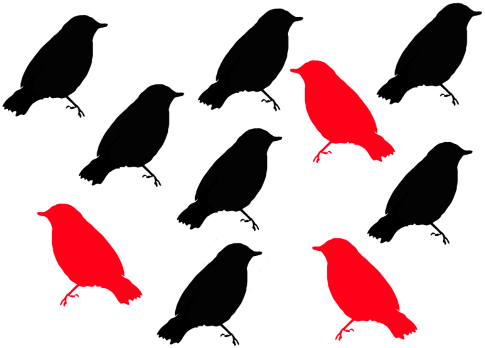
Population Protocols

Model of distributed computation
$\rightarrow$ to study systems of identical and anonymous *agents*:

- identical
- anonymous
- passively mobile
- tiny computational resources

(e.g. sensor networks or chemical systems)

Flock of Birds:



Question:  ≥ 4 **Goal:** Lasting Consensus

Population Protocols

Definition (Population Protocol)

A *population protocol* is a tuple $\mathcal{P} = (\mathrm{Q}, \mathcal{T}, \Sigma, \mathcal{I}, \mathcal{O})$ such that

- Q is a finite set of *states*,
- $\mathcal{T} \subseteq \bigcup_{2 \leq i \leq |\mathrm{Q}|} \mathrm{Q}^i \times \mathrm{Q}^i$ is a set of *transitions*,
- Σ is a non-empty finite input *alphabet*,
- $\mathcal{I} : \Sigma \to \mathrm{Q}$ is the *input function* and
- $\mathcal{O} : \mathrm{Q} \to \{0, 1\}$ is the *output function*.

Definition (Configuration)

A *configuration* of population protocol $\mathcal{P} = (\mathrm{Q}, \mathcal{T}, \Sigma, \mathcal{I}, \mathcal{O})$ is a multiset $C \in \mathbb{N}^{\mathrm{Q}}$ where $C(q)$ describes the number of agents in state $q \in \mathrm{Q}$.
The *output* of configuration C is

$$\mathcal{O}(C) = \begin{cases} b \in \{0,1\} & \text{if for all states } C(q) > 0 \Rightarrow \mathcal{O}(q) = b \\ \perp & \text{otherwise} \end{cases}$$

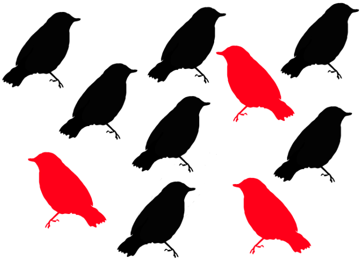
1. input: $x \in \mathbb{N}^{\Sigma}$
   $\downarrow$ input function $\mathcal{I}$ $\downarrow$
2. initial configuration: C_0
   $\downarrow$ transitions $\mathcal{T}$ $\downarrow$
3. fair[1] execution: $\sigma \overset{\text{def}}{=} C_0 \xrightarrow{t_1} C_1 \xrightarrow{t_2} C_2 \rightarrow \cdots$

$\mathcal{P}$ *computes* the predicate $\varphi : \mathbb{N}^{\Sigma} \rightarrow \{0, 1\}$, if for all inputs $x \in \mathbb{N}^{\Sigma}$ and corresponding fair executions $C_0 \xrightarrow{t_1} C_1 \xrightarrow{t_2} C_2 \rightarrow \cdots$ we reach the correct *lasting consensus*:

$$\exists i \in \mathbb{N} : \varphi(x) = \mathcal{O}(C_i) = \mathcal{O}(C_{i+1}) = \cdots$$

[1] A fair execution cannot avoid configurations forever.

Flock of Birds:



$$Q \stackrel{\text{def}}{=} \{0, 1, 2, 3, 4\}$$

$$\mathcal{T} \stackrel{\text{def}}{=} \{p, q \to \min(p+q, 4), 0 \mid p, q \in Q\} \cup \{p, 4 \to 4, 4 \mid p \in Q\}$$

$$\Sigma \stackrel{\text{def}}{=} \{\mathit{sick}, \mathit{healthy}\}$$

$$\mathcal{I}(x) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } x = \mathit{sick} \\ 0 & \text{if } x = \mathit{healthy} \end{cases}$$

$$\mathcal{O}(q) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } q = 4 \\ 0 & \text{otherwise} \end{cases}$$

Question:  ≥ 4

Question:

Is a given protocol correct?

$\rightarrow$ *TOWER*-hard [1] [2]

Goal: Automatic Verification

$\rightarrow$ need lower complexity!

$\rightarrow$ Blondin *et al.* [3]:
(incomplete) approach for *silent* protocols

$\rightarrow$ Peregrine

Definition (Silent Population Protocol)

A population protocol is *silent* if for every fair execution $C_0 \rightarrow C_1 \rightarrow \cdots$ there is a $i \in \mathbb{N}$ such that:

$$C_i = C_{i+1} = C_{i+2} = \cdots$$

Automatic Verification of non-silent Population Protocols

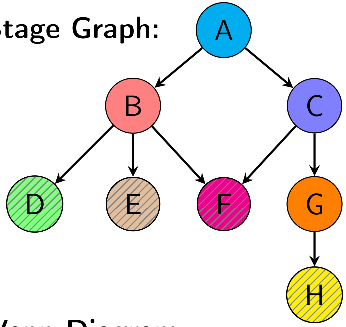
Termination Behaviour

silent protocols	vs	non-silent protocols
$\rightarrow$ reach terminal configuration $\rightarrow$ all transitions disabled $\rightarrow$ easy description / test		$\rightarrow$ reach lasting consensus BUT: How to describe "lasting"? $\rightarrow$ harder!

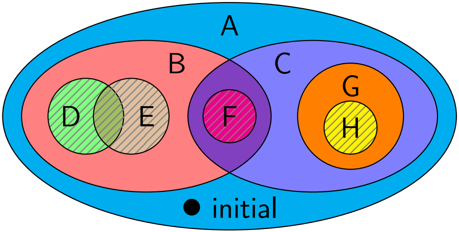
Idea: Group configurations into (infinite) sets
$\rightarrow$ Describe all fair executions at once!

Stage Graphs

Stage Graph:



Venn-Diagram:



Directed Acyclic Graph (DAG) of stages such that:

1. Stages are inductive sets of configurations.
   i.e. "can't leave"
2. Initial configurations are part of some stage.
3. non-terminal stage: Executions will enter substage.
4. terminal stage: correct consensus

Stage Graphs

Stage graphs are certificates for properties of the form:

$$\varphi_{pre} \Rightarrow FG\varphi_{post}$$

"If you start in a configuration that satisfies φ_{pre}, then you will eventually satisfy φ_{post} forever."

Theorem

Let Λ be a predicate. For $b \in \{0, 1\}$ let

$$\varphi_{init,b}(C) \stackrel{def}{=} \exists X \in \mathbb{N}^{\Sigma} : (\Lambda(X) = b) \wedge (\mathcal{I}(X) = C)$$

$$\varphi_{out,b}(C) \stackrel{def}{=} (\mathcal{O}(C) = b).$$

A population protocol $\mathcal{P}$ has a $(\varphi_{init,0}, \varphi_{out,0})$-stage-graph and a $(\varphi_{init,1}, \varphi_{out,1})$-stage-graph if and only if it computes the predicate Λ.

$\Rightarrow$ sound and complete

Stage Graphs

Proof.

"$\Rightarrow$":

1. Executions can't leave stages.
2. All executions start some stage.
3. Non-terminal & Fairness $\Rightarrow$ "enter" substage
4. Terminal $\Rightarrow$ correct consensus

"$\Leftarrow$": As protocol computes Λ, there are the needed stage graphs, each with 2 stages:

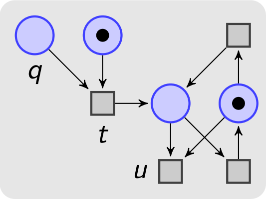
- Initial stage: all reachable configurations
- Terminal stage: all configurations with the correct lasting consensus

$\square$

Computing Stage Graphs

Idea: Protocols designed to work in stages
$\rightarrow$ correspond to non-reversible change in configuration:

- "death" of a transition
  Example: t and u are dead
  i.e. "t and u can't be enabled anymore."
- a state becomes "deserted"
  Example: q is deserted
  i.e. "q can't be populated anymore."



$\rightarrow$ automatically find such stages

Computing Stage Graphs

Stage Representation

Stage $S = (T_{dead}, Q_{deserted})$ where

- $T_{dead} \subseteq \mathcal{T}$ is the set of dead transitions.
- $Q_{deserted} \subseteq \mathrm{Q}$ is the set of deserted states.

Configuration C is in stage S if

1. there is a configuration $C_0 \models \varphi_{pre}$ such that $C_0 \xrightarrow{*} C$, and
2. T_{dead} are dead, and
3. $Q_{deserted}$ are deserted.

```
input:  protocol P = (Q, T, Σ, I, O)
        Presburger predicate φ_pre
        Presburger predicate φ_post

S_0 := (∅, ∅)
Unprocessed := {S_0}

while |Unprocessed| > 0
    S := Unprocessed.pop()

    if Substages(P, φ_pre, φ_post, S) fails
        then abort
    else
        Unprocessed := Unprocessed ∪ Substages(P, φ_pre, φ_post, S)
```

Computing Stage Graphs

Algorithm: Find new substages

```
input:  protocol P = (Q, T, Σ, I, O)
        Presburger predicate φ_pre
        Presburger predicate φ_post
        stage S = (T_dead, Q_deserted)

if Terminal(P, φ_pre, S, φ_post)
    return ∅

T'_dead := EventuallyDead(P, φ_pre, S)
if T'_dead ⊃ T_dead
    return {(T'_dead, Q_deserted)}

if Split(P, φ_pre, S) fails
    then abort
else return Split(P, φ_pre, S)
```

Parametric in 3 auxiliary functions

Terminal:
Try to prove: S is terminal

EventuallyDead:
Find "eventually dead" transitions

Split:
Split S in substages with more deserted states.

Computing Stage Graphs
Terminal

Need to decide: $C \in S$.
Problem: "reachable", "dead" and "deserted" are non-trivial

Idea: Overapproximate!

1. "reachable": use *potential reachability* [3]
   flow equation & siphons & traps
2. "dead": use "disabled"[2]
3. "deserted": use "empty"

Implementation: Use Z3 to check

$$\forall C : C \models \neg \mathrm{PotInStage}(\mathcal{P}, \varphi_{pre}, S) \vee \varphi_{post}$$

[2] We also use tighter approximations using the backwards coverability algorithm.

Computing Stage Graphs

EventuallyDead

Goal: Find transitions that will eventually become dead from every configuration $C \in S$.

Implementations:

- **Ranking function:**
  $\rightarrow$ imply eventual death of some transition
- **Layered termination:** [3]
  find "layer" $L \subseteq \mathcal{T}$ and ranking function such that
  - L will eventually be disabled, and
  - $\mathit{Disabled}(L) \Rightarrow \mathit{Dead}(L)$
- **Combined:**
  use ranking functions and layered termination

Computing Stage Graphs

Split

Goal: Split stage into substages with more deserted states.
(i.e. "case distinction")

Idea: empty siphon $\Rightarrow$ deserted

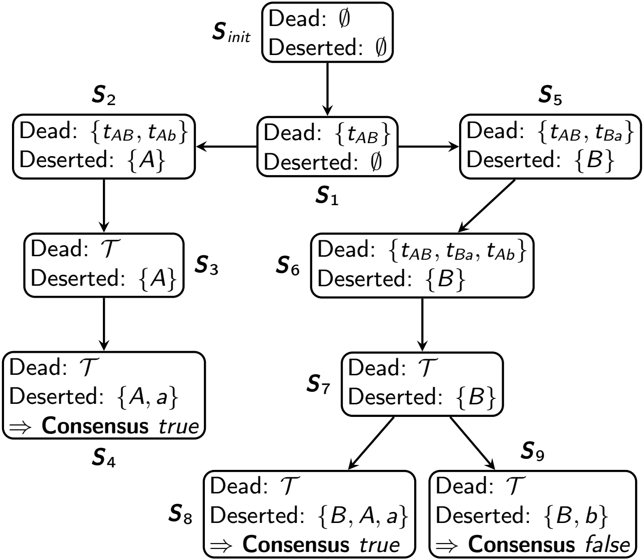
$\rightarrow$ find set of siphons R such that

$$\forall C : C \models \neg \mathrm{PotInStage}(\mathcal{P}, \varphi_{pre}, S) \vee \bigvee_{R_i \in R} empty(R_i)$$

Implementation: Guess siphons using Z3.

Computing Stage Graphs

Example



Majority Protocol
"$A \leq B$"

$t_{AB} : AB \to ab$

$t_{Ab} : Ab \to Aa$

$t_{Ba} : Ba \to Bb$

$t_{ab} : ab \to bb$

protocol	predicate	silent	$\lvert Q \rvert$	$\lvert \mathcal{T} \rvert$	proven	time
Majority	$A \leq B$	yes	4	4	yes	< 1s
A&C(11,9)	$A \leq B$	no	28	406	yes	700s
Flock-of-Birds	$X \geq 60$	yes	61	1891	yes	328s
succinct FoB.	$X \geq 2^{35} - 1$	yes	70	1294	yes	334s
suc. rev. FoB.	$X \geq 63$	no	12	31	yes	40s
Remainder	$\sum_{1 \leq i < 20} i \cdot x_i \equiv_{20} 0$	yes	22	250	yes	565s
succinct Rem.	$\sum_{1 \leq i < 63} i \cdot x_i \equiv_{63} 0$	no	16	41	yes	75s
Threshold	$-2a - b + c + 2d < 3$	yes	36	495	yes	32s
succinct Thr.	$-2a - b + c + 2d < 63$	yes	20	66	yes	100s

Table: Automatic verification of silent and non-silent protocols using stage graphs.

Computing Stage Graphs

Results: Leader election

We can even verify leader election! (i.e. via postcondition)

protocol	n	silent	$\lvert Q \rvert$	$\lvert \mathcal{T} \rvert$	proven	time
simple	∞	yes	2	1	yes	< 1s
Israeli-Jalfon	70	no	140	280	yes	2537s
Herman	91	no	182	182	no	203s
Herman modified	91	no	182	182	yes	2785s

Table: Automatic verification of leader election protocols for *n* agents.

Termination Time

Important questions in practise:

- Correctness: ✓
- Fast: ?

But what does "fast" mean?

→ expected number of interactions

→ probabilistic model

(i.e. "random" instead of fairness)

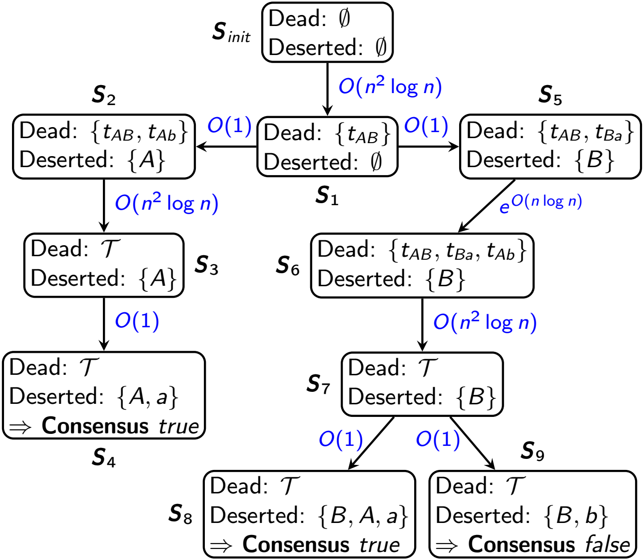
Apply idea of Blondin *et al.* [4]!

Let $n = |C_0|$.

- **Terminal:** $O(1)$
- **Split:** $O(1)$
- **EventuallyDead:**
  - layered: $O(n^n)$
  - ranking: $O(n^c)$ for some constant c
  - layered + ranking: $O(n^3)$
  - layered + ranking + "fast": $O(n^2 \log n)$

Termination Time

Example



Majority Protocol
"$A \leq B$"

$t_{AB} : AB \to ab$

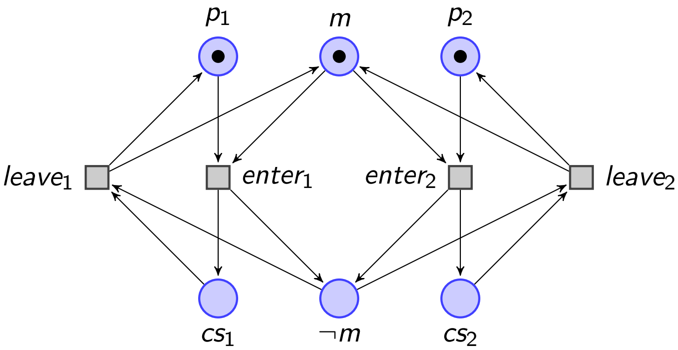
$t_{Ab} : Ab \to Aa$

$t_{Ba} : Ba \to Bb$

$t_{ab} : ab \to bb$

protocol	$\lvert Q\rvert$	$\lvert\mathcal{T}\rvert$	bound	time
Majority	4	4	$O(n^n)$	< 1s
simple leader election	2	1	$O(n^2 \log n)$	< 1s
Flock-of-Birds(45)	46	2026	$O(n^3)$	307s
succinct FoB(511)	18	97	$O(n^3)$	2.5s
suc. rev. FoB(63)	12	31	$O(n^c)$	307s
Remainder($\equiv_4$)	6	18	$O(n^2 \log n)$	2.8s
Threshold(< 2)	28	301	$O(n^3)$	62s
A&C(7,1)	10	55	$O(n^2 \log n)$	8.3s
A&C(11,10)	32	528	$O(n^3)$	550s

Table: Automatically found and proven speed bounds.



Question: Can we verify liveness?
E.g. will process 1 enter its critical section infinitely often?

Answer: No!
$\rightarrow$ property does not have form $\varphi_{pre} \Rightarrow FG\varphi_{post}$

LTL
Overview

liveness property (LTL)

$GF(enter_1)$

φ

negate

$FG(\neg enter_1)$

$\neg\varphi$

transform

population protocol

LDBA B $\times$ $\mathcal{P}$

product

product protocol

$\mathcal{P}'$

verify:
B can't accept

✓

We can verify liveness of a single process in mutex algorithms!

Mutex algorithm	processes	proven	time
Simple	400	yes	2049s
Array	11	yes	2284s
Burns	6	yes	1074s
Peterson	2	yes	< 1s
Dijkstra	4	yes	3221s
Szymanski	3	yes	38s
Lehmann Rabin	10	yes	3141s

Table: Automatic verification of liveness of a single process in mutex algorithms.

Future Work

Verify more expressive models?

- Petri nets with inhibitor arcs
- population protocols with broadcast
- $\cdots$

$\rightarrow$ automatically?

Other fairness assumptions?

References I

[1] Javier Esparza, Pierre Ganty, Jérôme Leroux, and Rupak Majumdar.
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[2] Wojciech Czerwiński, Sławomir Lasota, Ranko Lazić, Jérôme Leroux, and Filip Mazowiecki.
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[3] Michael Blondin, Javier Esparza, Stefan Jaax, and Philipp J Meyer.
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References II

[4] Michael Blondin, Javier Esparza, and Antonín Kucera.
Automatic analysis of expected termination time for population protocols.
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