Fast and Succinct Population Protocols for Presburger Arithmetic

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Technical University of Munich

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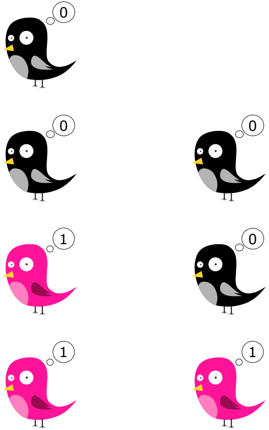
Introduction to Population Protocols

Population Protocols = model of computation



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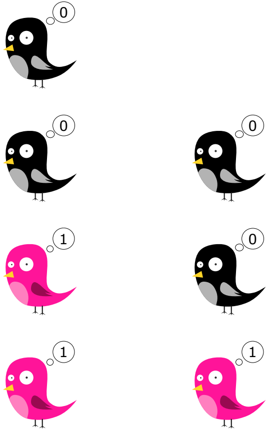

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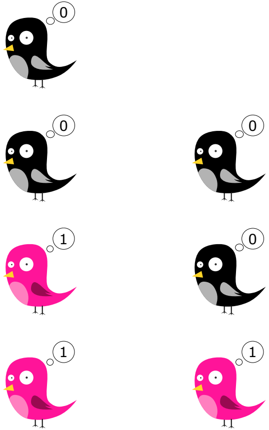

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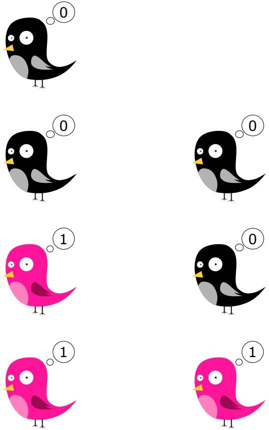

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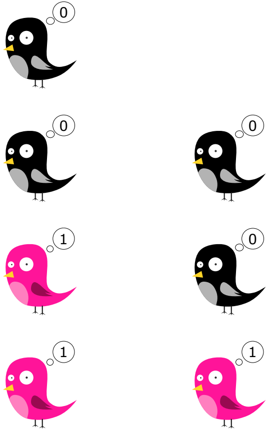

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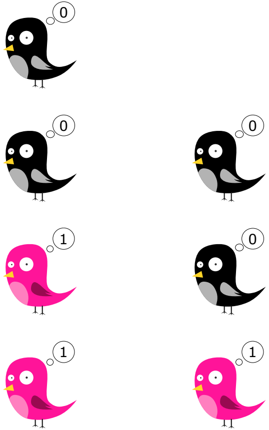

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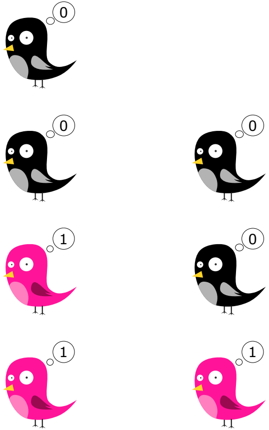

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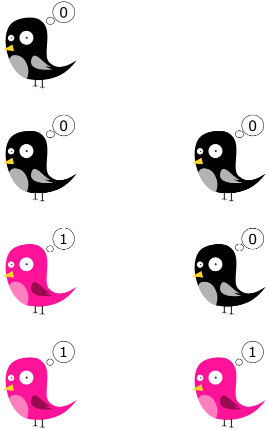

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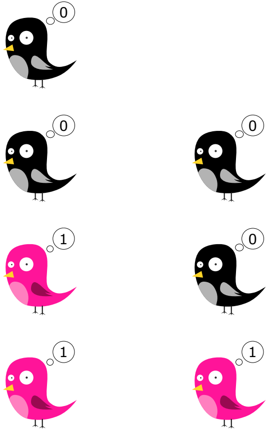

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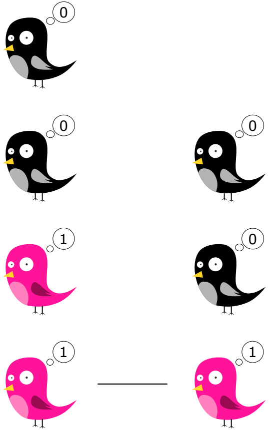

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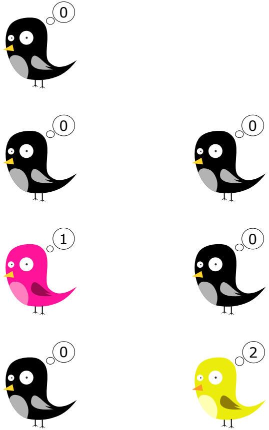

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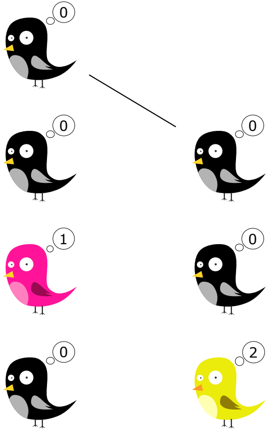

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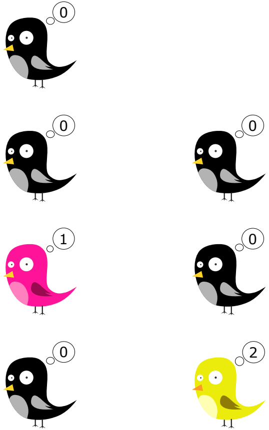

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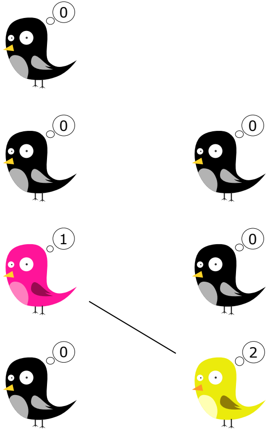

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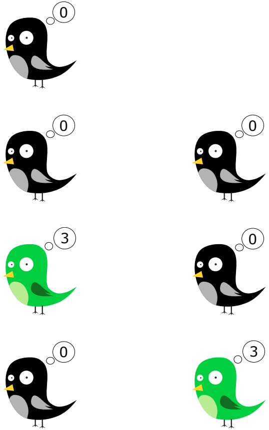

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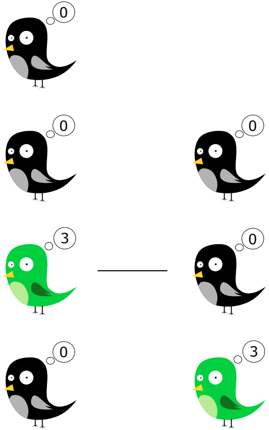

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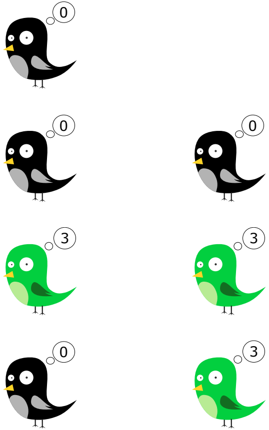

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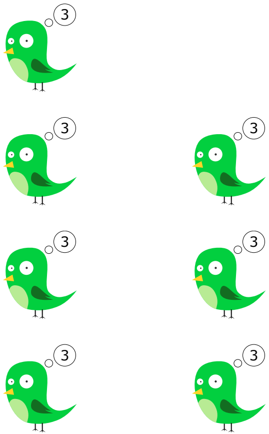

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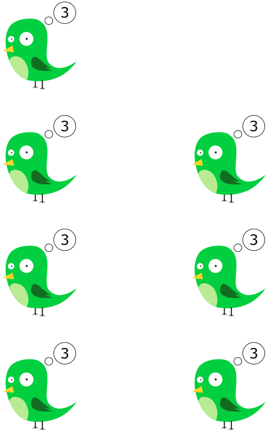

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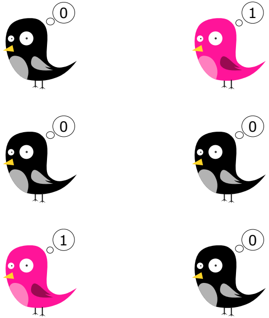

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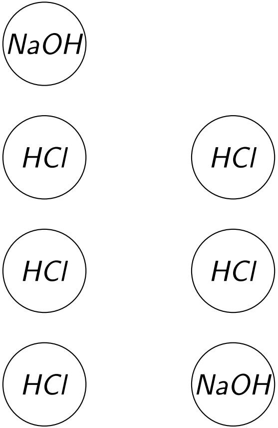

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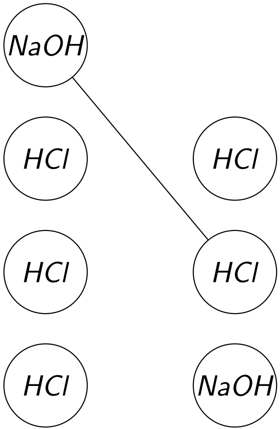

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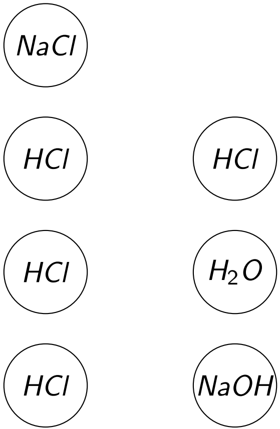
State complexity: # species.



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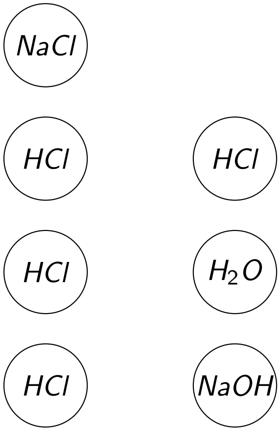
Accordingly for protocols:
$|\{0, 1, 2, 3\}| = 4$.



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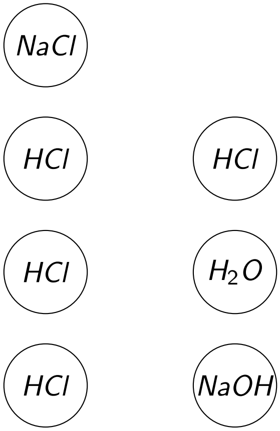
Mobile sensor networks, ...



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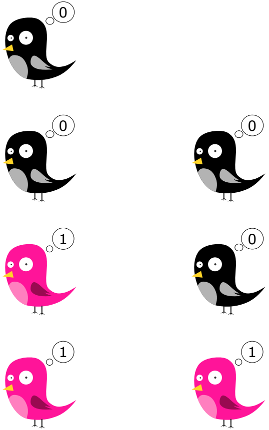

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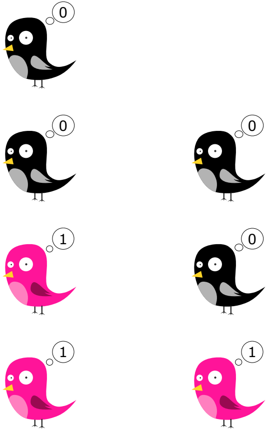

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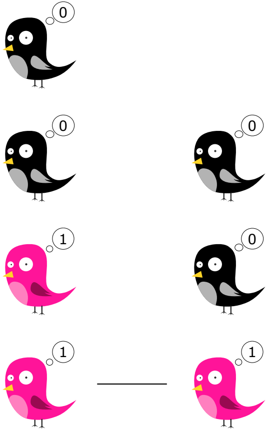

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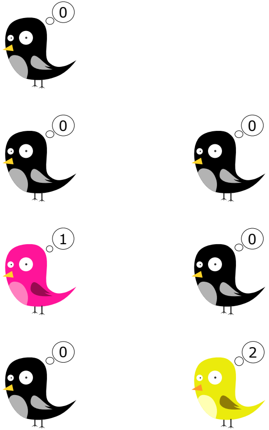

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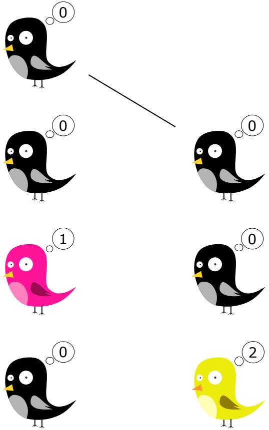

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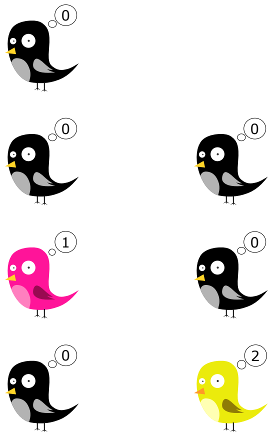

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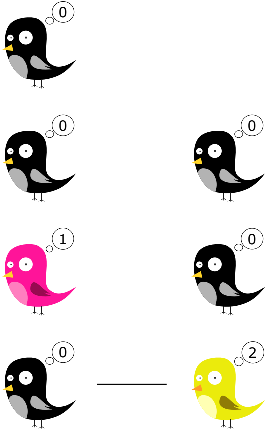

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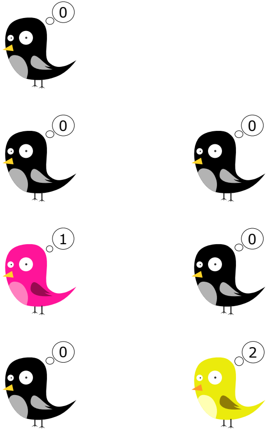

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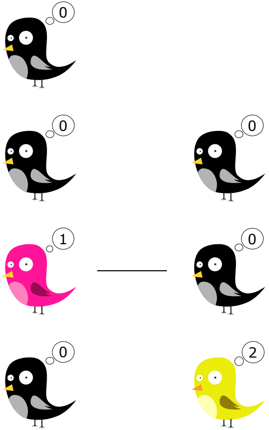

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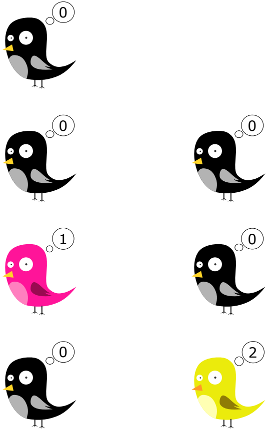
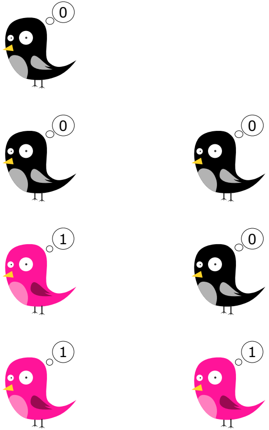


Expressive Power

Special classes of properties.



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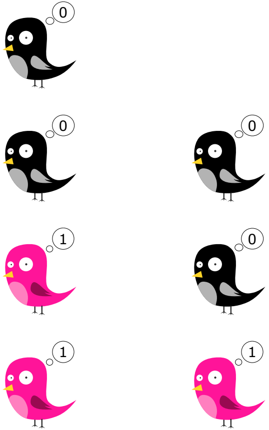

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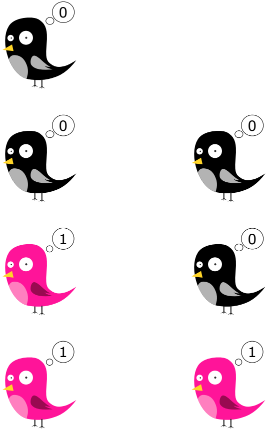

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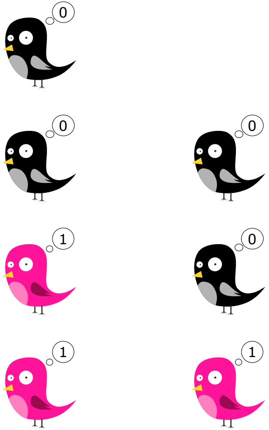

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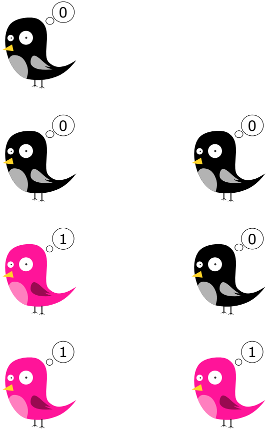

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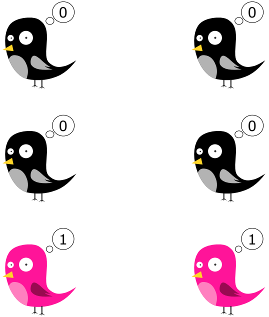

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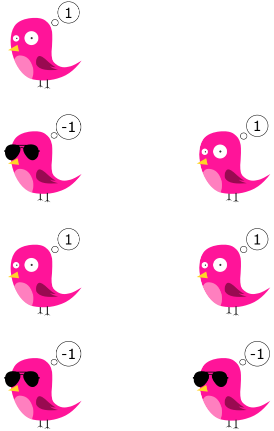
Glasses $=$ Negative value



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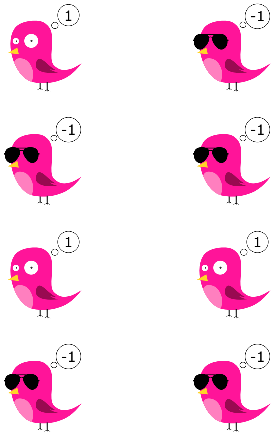

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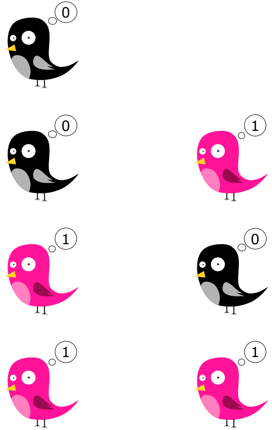

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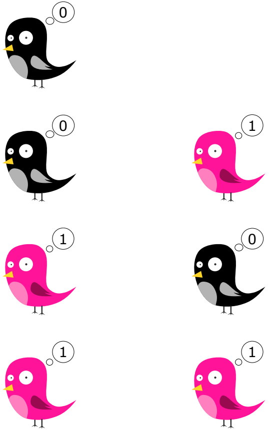

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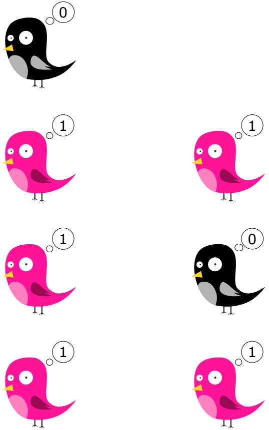

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Allowed initial states: $4, -3$. Decide ≥ 0.
$4x - 3y \geq 0$.

Angluin et. al. [2006]: Expressive power: Exactly all boolean combinations of threshold and modulo. This class is called Quantifier Free Presburger Arithmetic (*QFPA*).

Expressive Power

Special classes of properties.

Class 1 (Threshold):
Every bird: Initially integer value
Decide total sum $\geq c$.

Class 2 (Modulo):
Every bird: Initially integer value
Decide total sum $\equiv_m c$

Regarding $|\varphi|$: Encode predicates.

Allowed initial states: $4, -3$. Decide ≥ 0.
$4x - 3y \geq 0$.

$|\varphi|=$ length of string with numbers in binary.

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Goal: Synthesis Procedure

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Procedure for following problem:

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Input: Formula $\varphi \in \mathit{QFPA}$.

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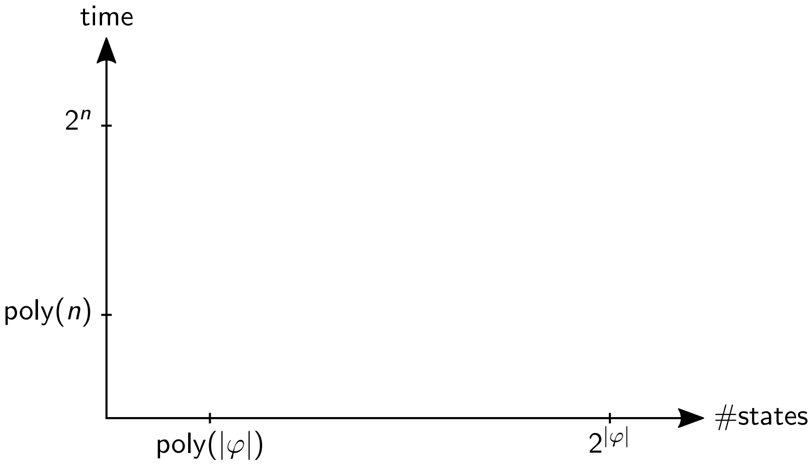
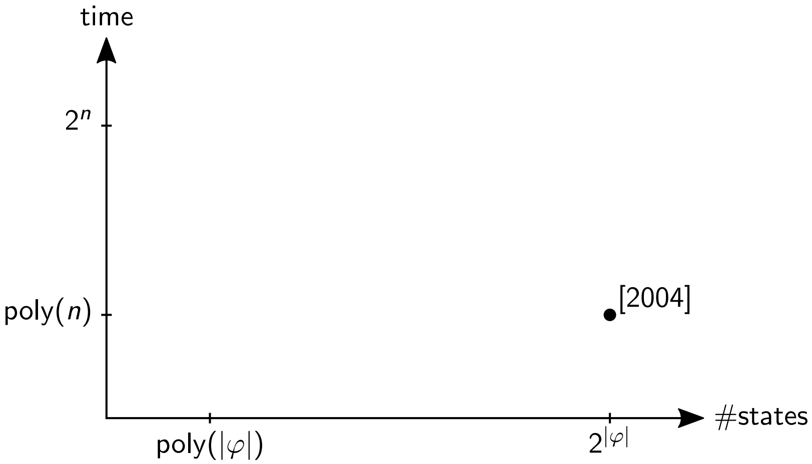
Synthesis procedures are compared via

- state complexity of protocols in $|\varphi|$,
- speed in $n := \#$agents participating.

Prior Work



Prior Work



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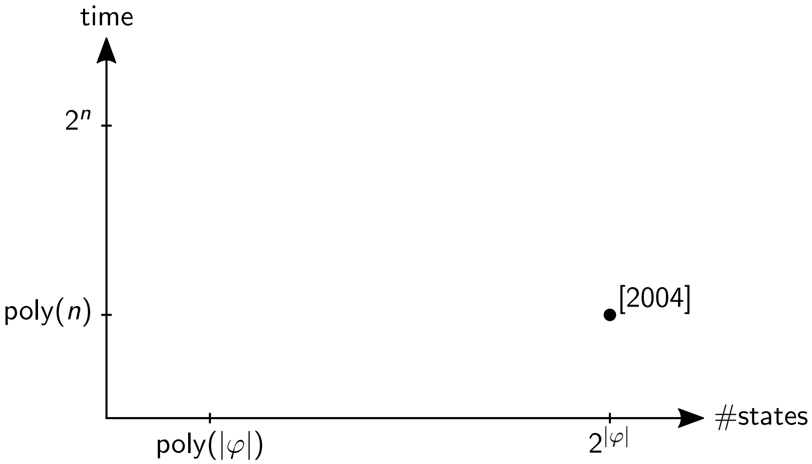

$c + 1$ states for $x \geq c$ is exponential in $|\varphi|$.

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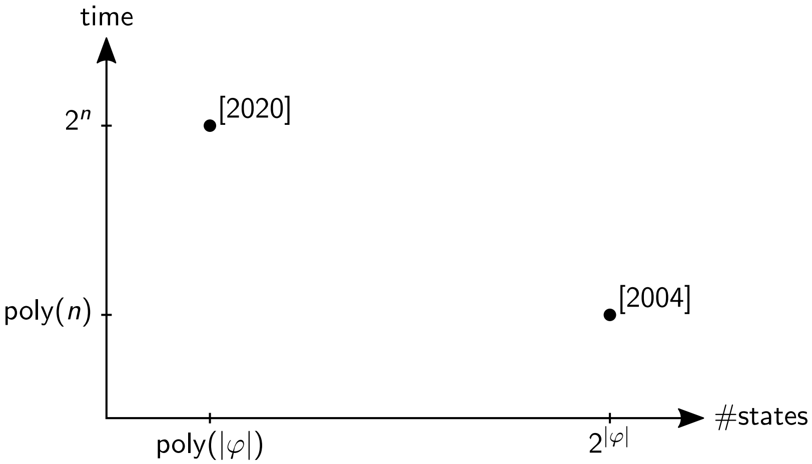

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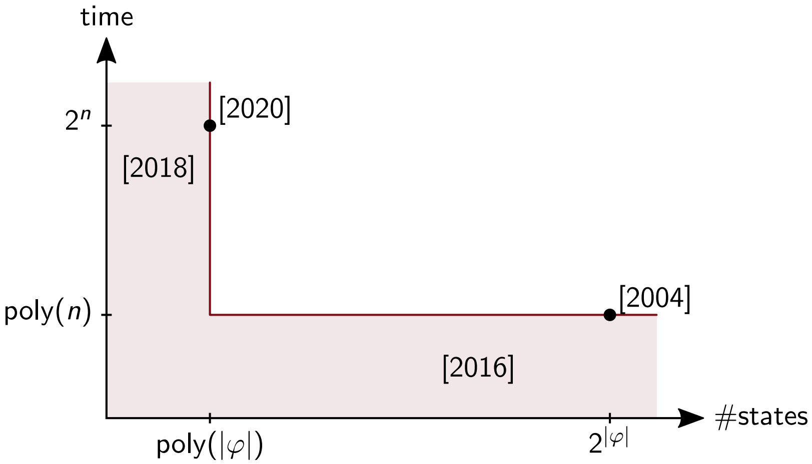

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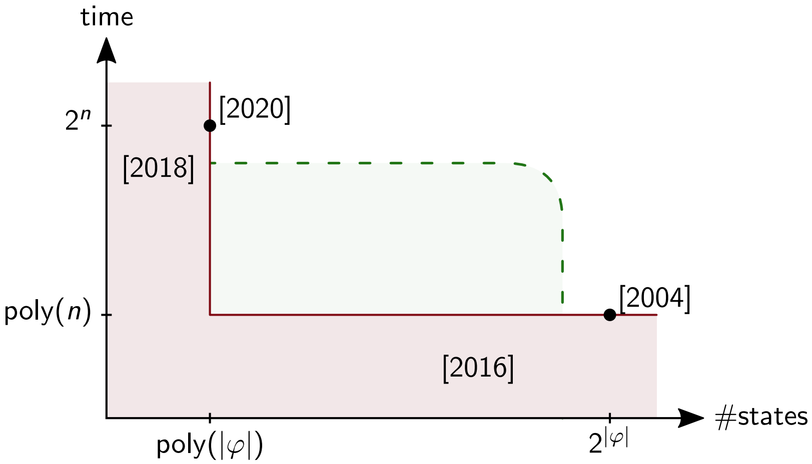

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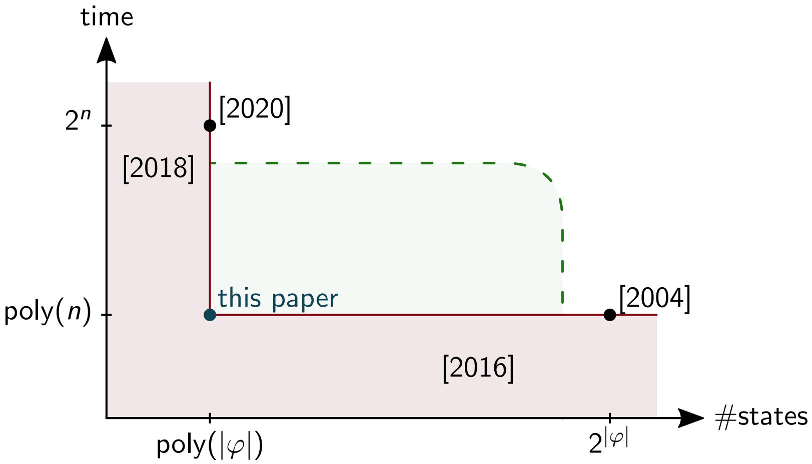
Overview



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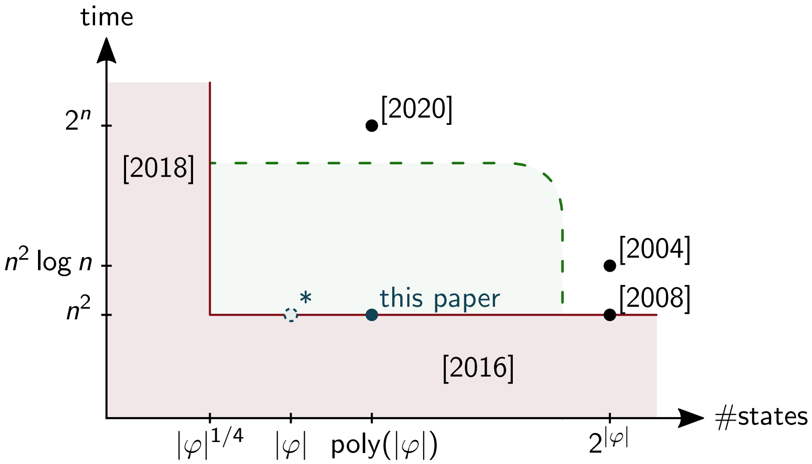

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$*$: $n \in \Omega(|\varphi|)$

Roadmap towards Fast and Succinct Population Protocols

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- To simplify protocol design, we introduce a more general model.

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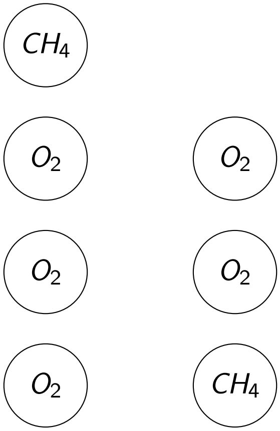
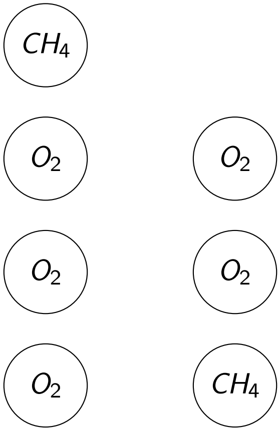
- Design succinct PCs satisfying a simple property.
- Convert them to population protocols.

Extension 1: Multiway interactions



Extension 1: Multiway interactions

Reminder: Chemical reaction networks.



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$$CH_4 + 2 \cdot O_2 \rightarrow CO_2 + 2 \cdot H_2O$$
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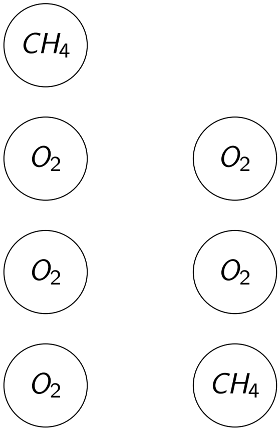

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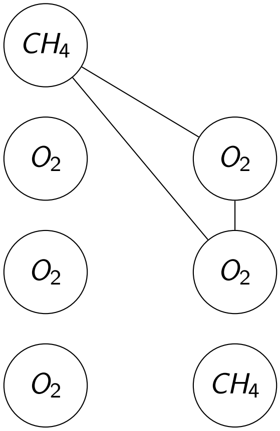

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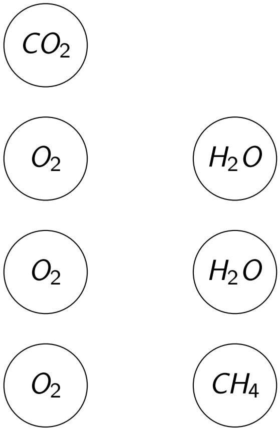

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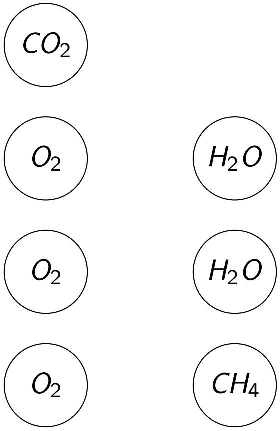

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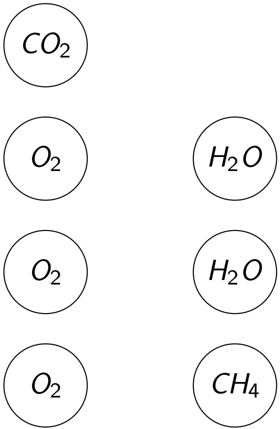
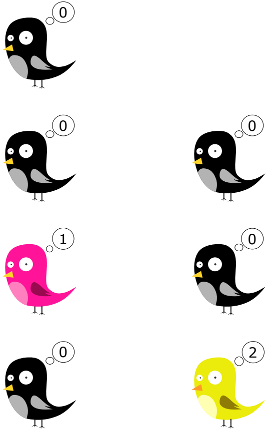
Chemical reactions often have only few types of reactants.
We only allow multiways with two types of reacting states.



Extension 2: Output Function



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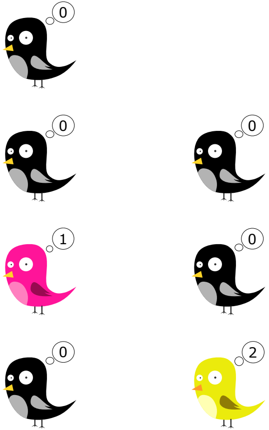

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i, j &\mapsto i + j, 0 && \text{if } i + j < 3, \\
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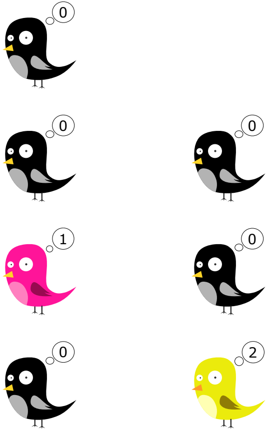

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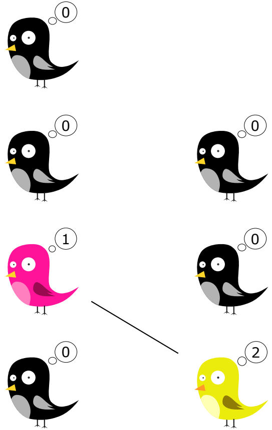

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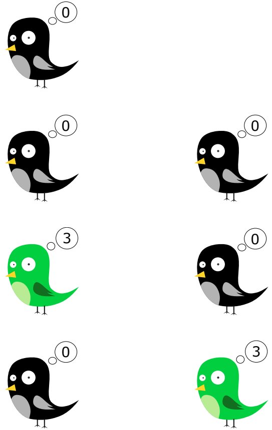

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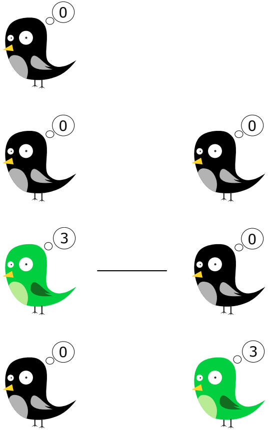

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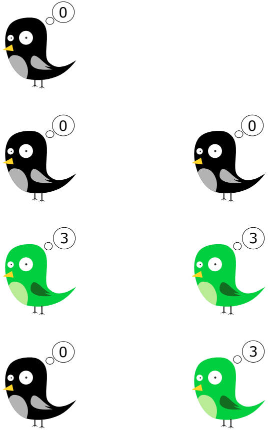

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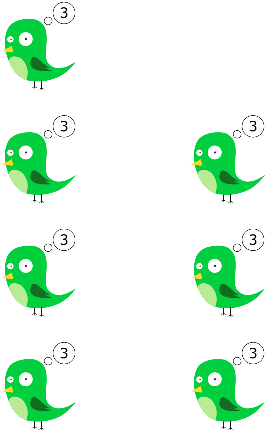

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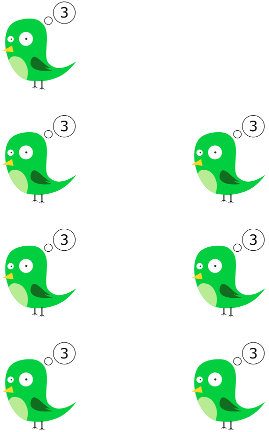
Output broadcast has little in common with rest of the protocol.



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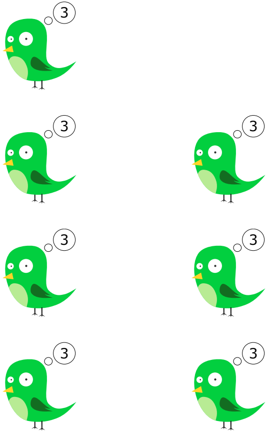
Split these two parts.



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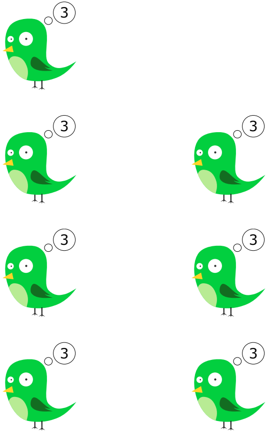
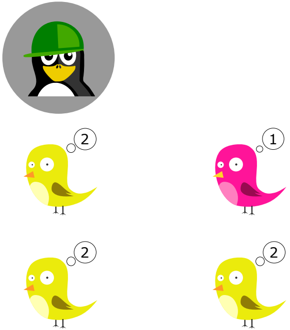
Output broadcast has little in common with rest of the protocol.

Split these two parts.

More general output function.

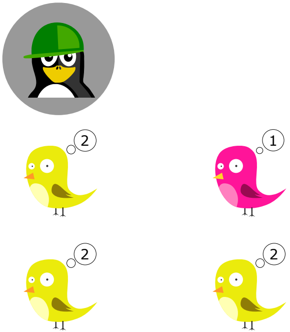


Extension 3: Helpers



Extension 3: Helpers

Auxiliary agents which do not count towards the input.



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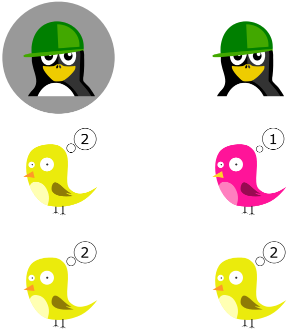

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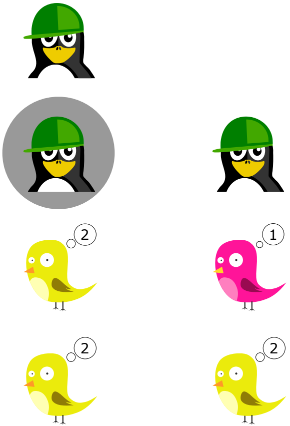
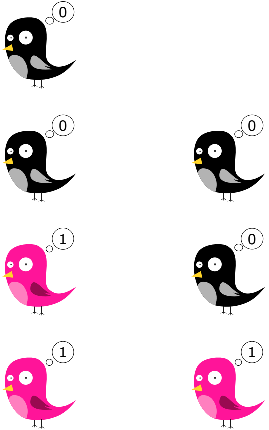
Idea: Computations often require auxiliary variables/gadgets.




0
0
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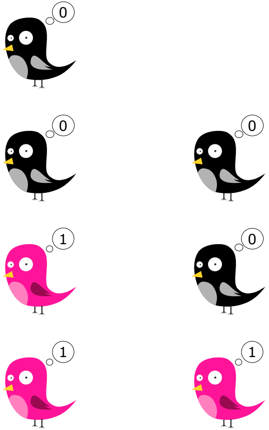
To ensure speed, we need bounded computers.



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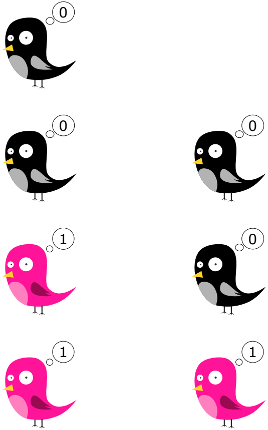

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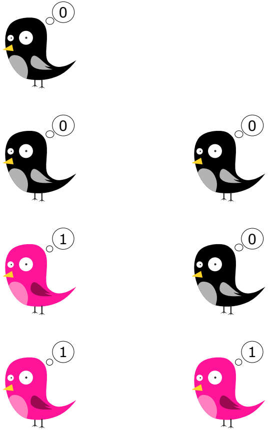

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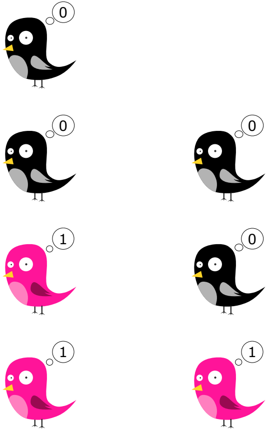

Conversion of Population Computers/Main Theorems

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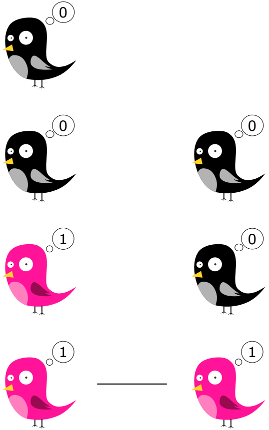

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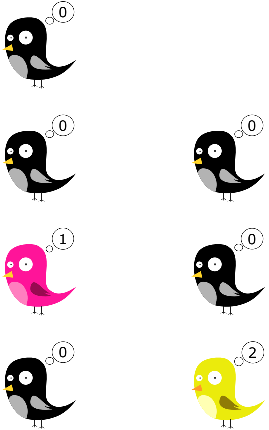

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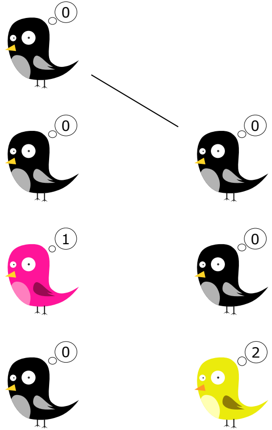

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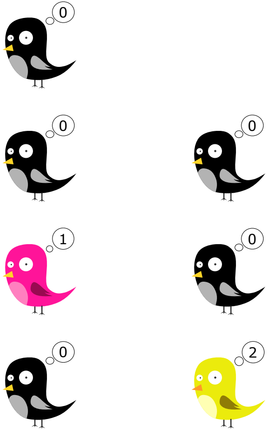

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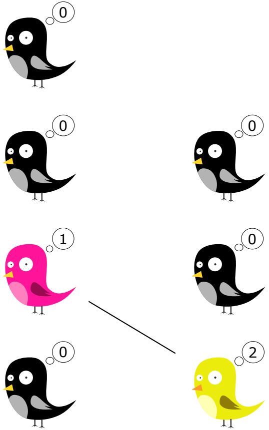

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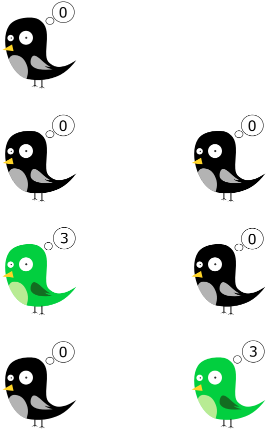

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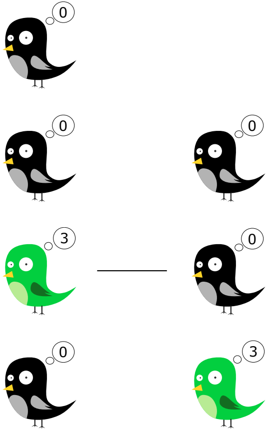

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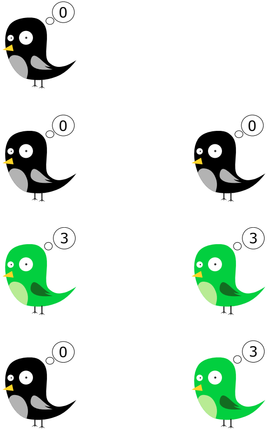

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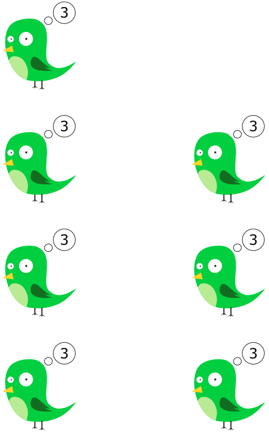

Conversion of Population Computers/Main Theorems

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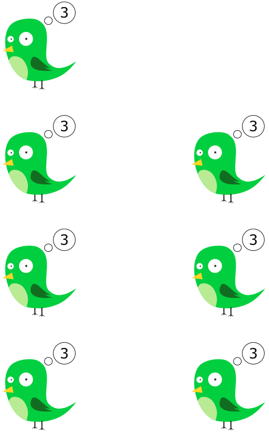
Determining boundedness does not require
a complicated analysis.



Population Computer

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Bounded

Conversion of Population Computers/Main Theorems

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Bounded

$\rightarrow\rightarrow\rightarrow$

Population Computer		Population Protocol
State complexity $\mathcal{O}(\lvert\varphi\rvert)$	$\to\to\to$	
Bounded		

Conversion of Population Computers/Main Theorems

Population Computer		Population Protocol
State complexity $\mathcal{O}(\vert\varphi\vert)$	$\to\to\to$	State complexity $\mathcal{O}(\vert\varphi\vert^2)$
Bounded		

Conversion of Population Computers/Main Theorems

Population Computer		Population Protocol
State complexity $\mathcal{O}(\|\varphi\|)$	$\rightarrow\rightarrow\rightarrow$	State complexity $\mathcal{O}(\|\varphi\|^2)$
Bounded		Speed $\mathcal{O}(n^3)$

Conversion of Population Computers/Main Theorems

Population Computer		Population Protocol
State complexity $\mathcal{O}(\lvert\varphi\rvert)$	$\rightarrow\rightarrow\rightarrow$	State complexity $\mathcal{O}(\lvert\varphi\rvert^2)$
Bounded		Speed $\mathcal{O}(n^3)$
		Inputs fulfilling $n \in \Omega(\lvert\varphi\rvert)$

Conversion of Population Computers/Main Theorems

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Bounded

$\rightarrow\rightarrow\rightarrow$

Population Protocol

State complexity $\mathcal{O}(|\varphi|^2)$

Speed $\mathcal{O}(n^3)$

Inputs fulfilling $n \in \Omega(|\varphi|)$

Population Computer

Conversion of Population Computers/Main Theorems

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Bounded

$\rightarrow\rightarrow\rightarrow$

Population Protocol

State complexity $\mathcal{O}(|\varphi|^2)$

Speed $\mathcal{O}(n^3)$

Inputs fulfilling $n \in \Omega(|\varphi|)$

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Conversion of Population Computers/Main Theorems

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Bounded

$\rightarrow\rightarrow\rightarrow$

Population Protocol

State complexity $\mathcal{O}(|\varphi|^2)$

Speed $\mathcal{O}(n^3)$

Inputs fulfilling $n \in \Omega(|\varphi|)$

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Rapid

Conversion of Population Computers/Main Theorems

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Bounded

$\rightarrow\rightarrow\rightarrow$

Population Protocol

State complexity $\mathcal{O}(|\varphi|^2)$

Speed $\mathcal{O}(n^3)$

Inputs fulfilling $n \in \Omega(|\varphi|)$

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Rapid

$\rightarrow\rightarrow\rightarrow$

Conversion of Population Computers/Main Theorems

Population Computer		Population Protocol
State complexity $\mathcal{O}(\lvert\varphi\rvert)$	$\longrightarrow\!\longrightarrow\!\longrightarrow$	State complexity $\mathcal{O}(\lvert\varphi\rvert^2)$
Bounded		Speed $\mathcal{O}(n^3)$
		Inputs fulfilling $n \in \Omega(\lvert\varphi\rvert)$

Population Computer		Population Protocol
State complexity $\mathcal{O}(\lvert\varphi\rvert)$	$\longrightarrow\!\longrightarrow\!\longrightarrow$	
Rapid		

Conversion of Population Computers/Main Theorems

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Bounded

$\rightarrow\rightarrow\rightarrow$

Population Protocol

State complexity $\mathcal{O}(|\varphi|^2)$

Speed $\mathcal{O}(n^3)$

Inputs fulfilling $n \in \Omega(|\varphi|)$

Population Computer

State complexity $\mathcal{O}(|\varphi|)$

Rapid

$\rightarrow\rightarrow\rightarrow$

Population Protocol

State complexity $\mathcal{O}(|\varphi|)$

Conversion of Population Computers/Main Theorems

Population Computer		Population Protocol
State complexity $\mathcal{O}(\lvert\varphi\rvert)$	$\to\to\to$	State complexity $\mathcal{O}(\lvert\varphi\rvert^2)$
Bounded		Speed $\mathcal{O}(n^3)$
		Inputs fulfilling $n \in \Omega(\lvert\varphi\rvert)$

Population Computer		Population Protocol
State complexity $\mathcal{O}(\lvert\varphi\rvert)$	$\to\to\to$	State complexity $\mathcal{O}(\lvert\varphi\rvert)$
Rapid		Speed $\mathcal{O}(n^2)$

Conversion of Population Computers/Main Theorems

Population Computer		Population Protocol
State complexity $\mathcal{O}(\lvert\varphi\rvert)$	$\to\to\to$	State complexity $\mathcal{O}(\lvert\varphi\rvert^2)$
Bounded		Speed $\mathcal{O}(n^3)$
		Inputs fulfilling $n \in \Omega(\lvert\varphi\rvert)$

Population Computer		Population Protocol
State complexity $\mathcal{O}(\lvert\varphi\rvert)$	$\to\to\to$	State complexity $\mathcal{O}(\lvert\varphi\rvert)$
Rapid		Speed $\mathcal{O}(n^2)$
		Inputs fulfilling $n \in \Omega(\lvert\varphi\rvert)$

Conversion of Population Computers/Main Theorems

Population Computer		Population Protocol
State complexity $\mathcal{O}(\lvert\varphi\rvert)$	$\rightarrow\rightarrow\rightarrow$	State complexity $\mathcal{O}(\lvert\varphi\rvert^2)$
Bounded		Speed $\mathcal{O}(n^3)$
		Inputs fulfilling $n \in \Omega(\lvert\varphi\rvert)$

Population Computer		Population Protocol
State complexity $\mathcal{O}(\lvert\varphi\rvert)$	$\rightarrow\rightarrow\rightarrow$	State complexity $\mathcal{O}(\lvert\varphi\rvert)$
Rapid		Speed $\mathcal{O}(n^2)$
		Inputs fulfilling $n \in \Omega(\lvert\varphi\rvert)$

Blondin et. al. [2020]: Remove input restriction at cost of $\mathcal{O}(\text{poly}(\lvert\varphi\rvert))$ states.

Thank you for your attention!