Decision Power of Weak Asynchronous Models of Distributed Computing

Philipp Czerner Roland Guttenberg Javier Esparza Martin Helfrich

Technische Universität München

Common Ground and Differing Aspects

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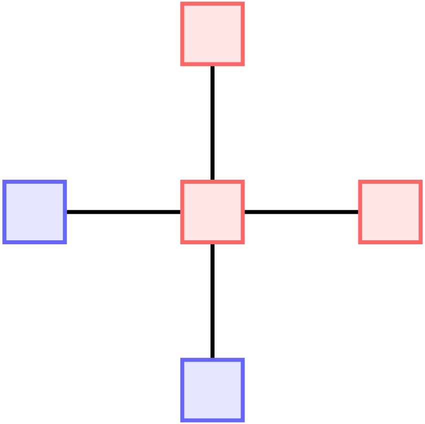

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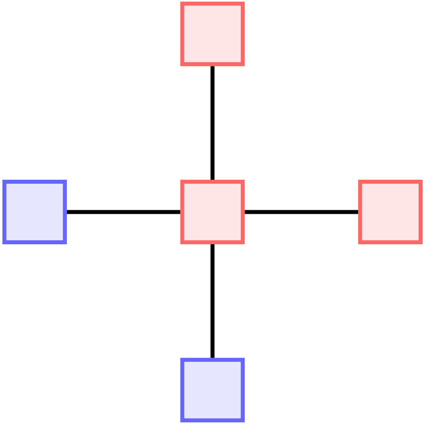

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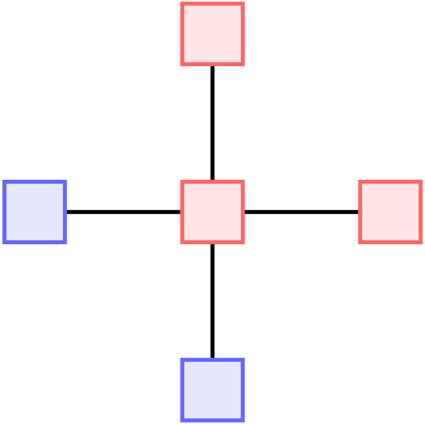

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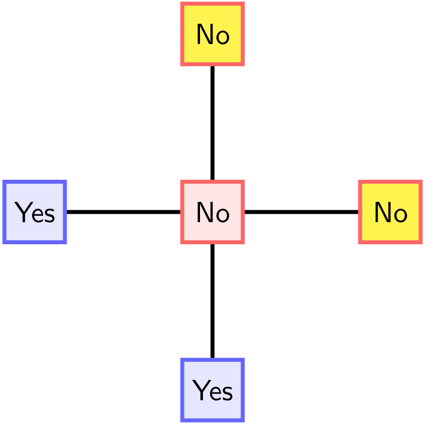

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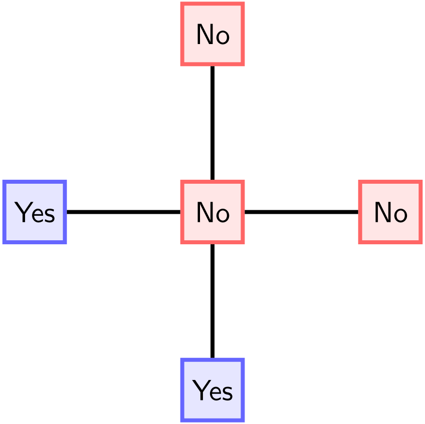

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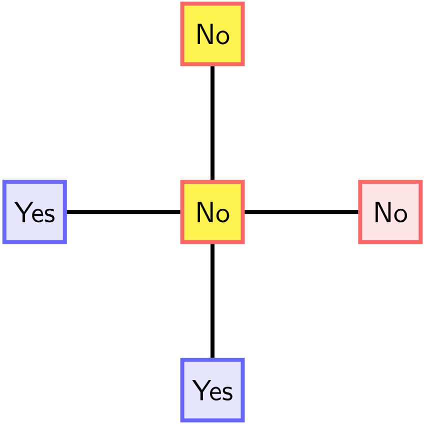

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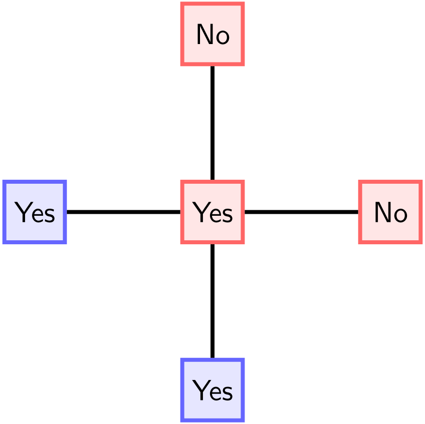

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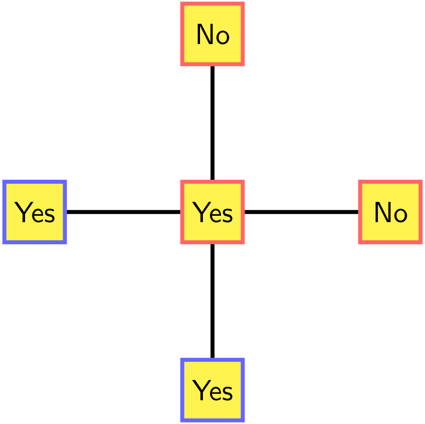

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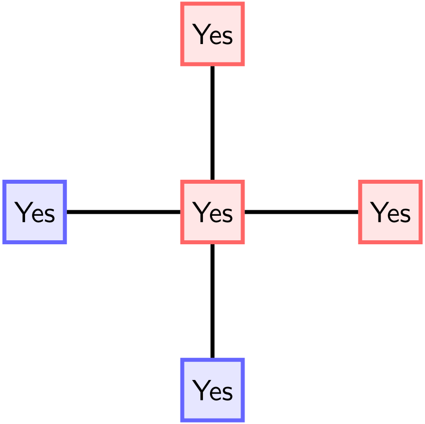

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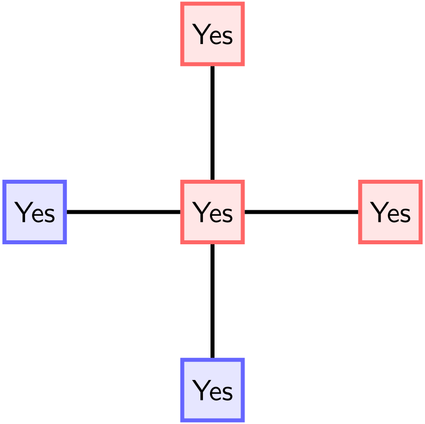

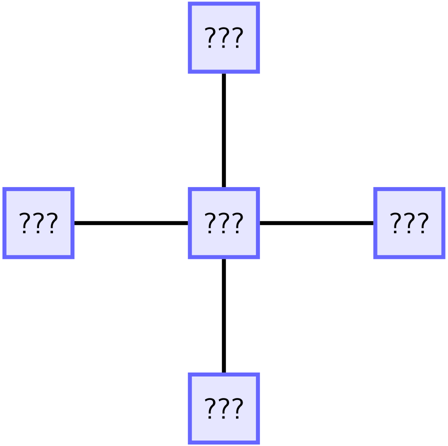

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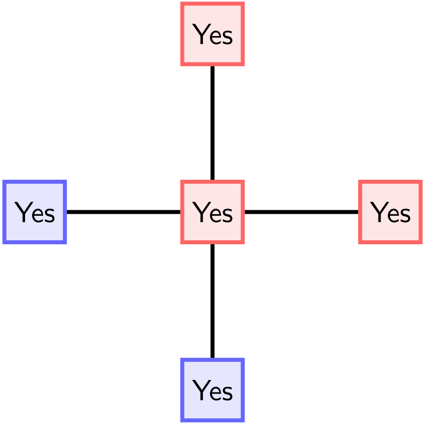

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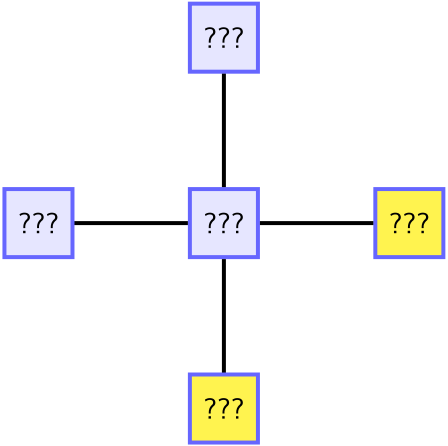

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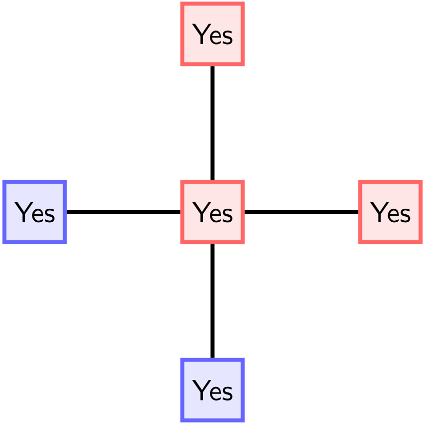

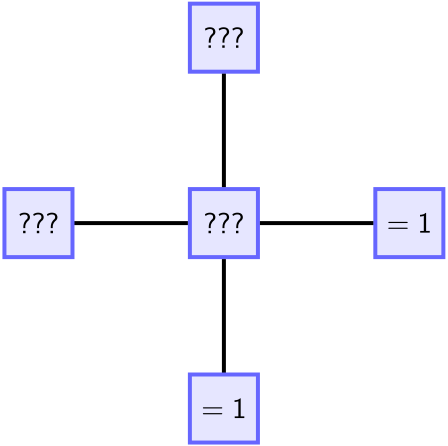

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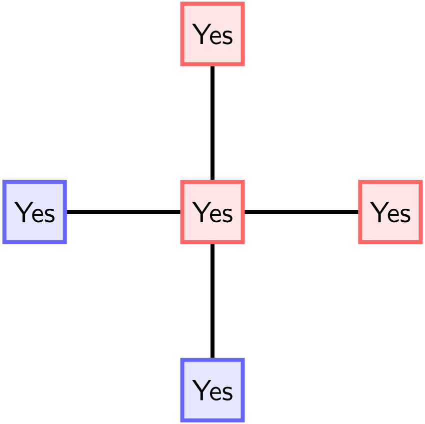

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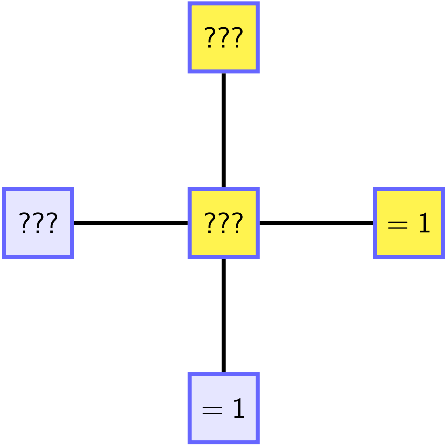

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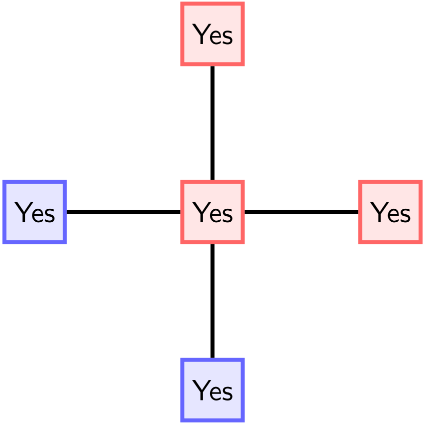

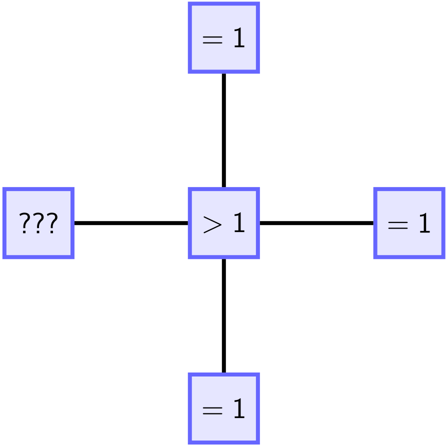

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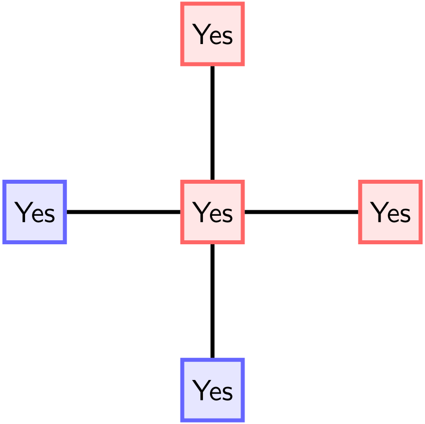

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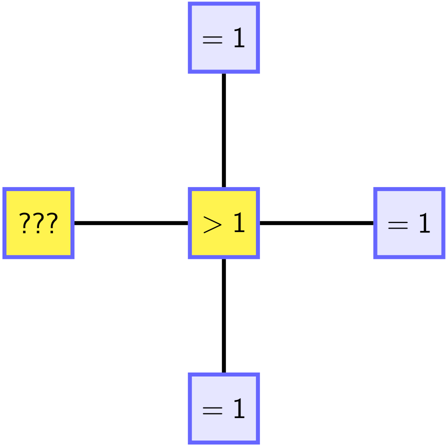

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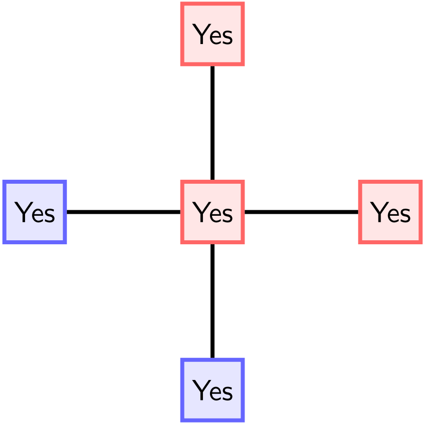

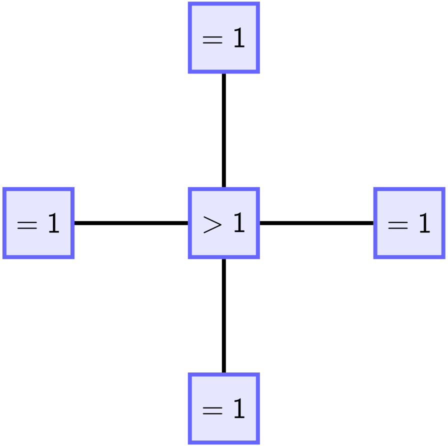

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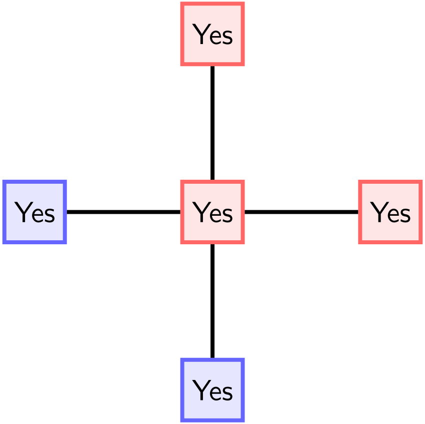

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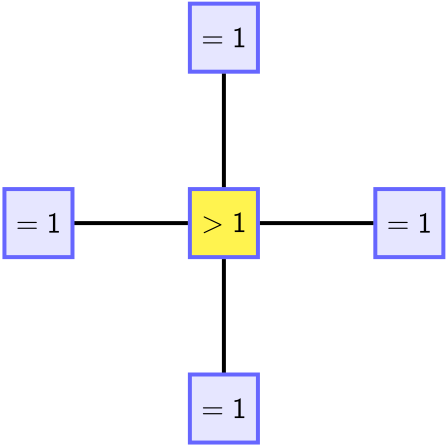

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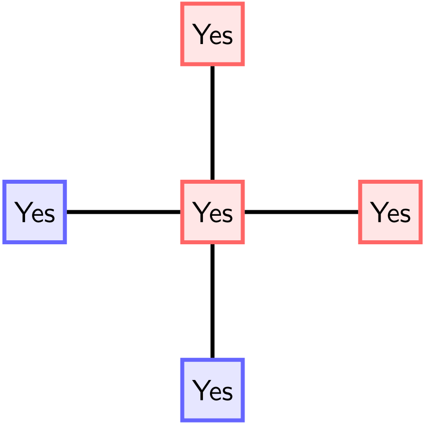

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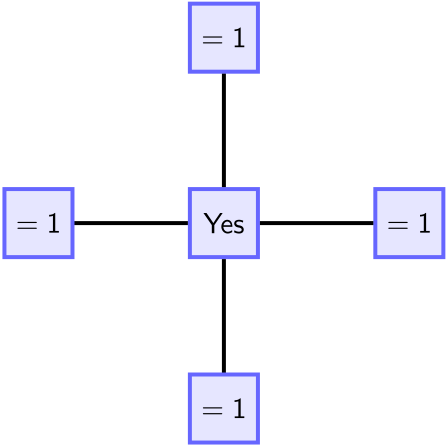

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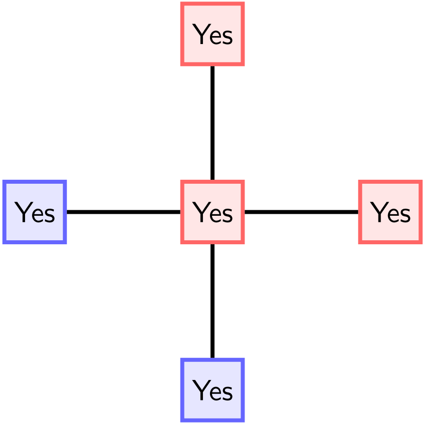

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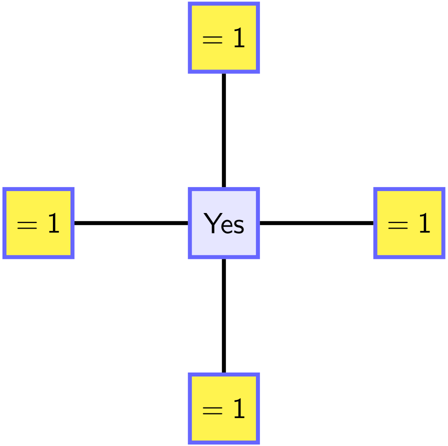

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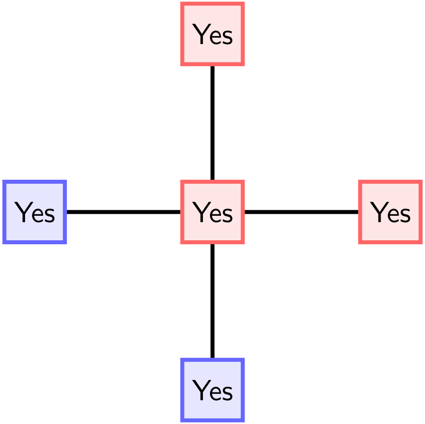

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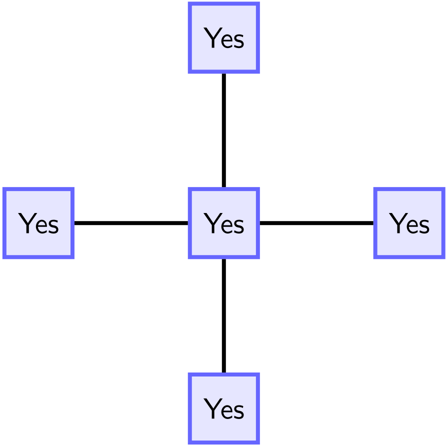
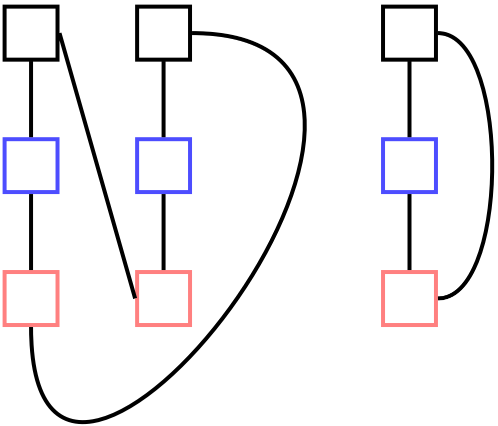


Nodes had to count their neighbors.

Distinguish Cycles(?)

Is it possible to distinguish different length cycles?



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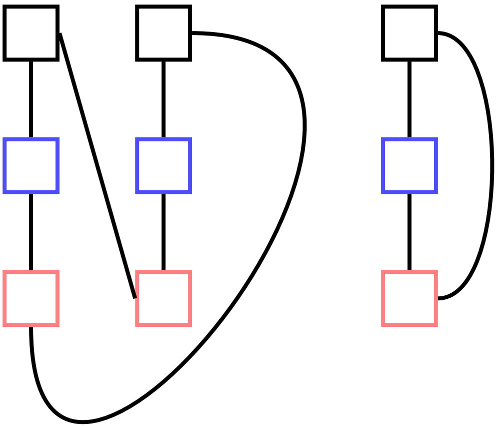

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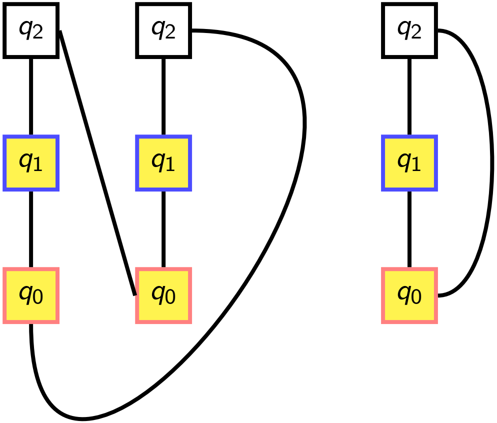

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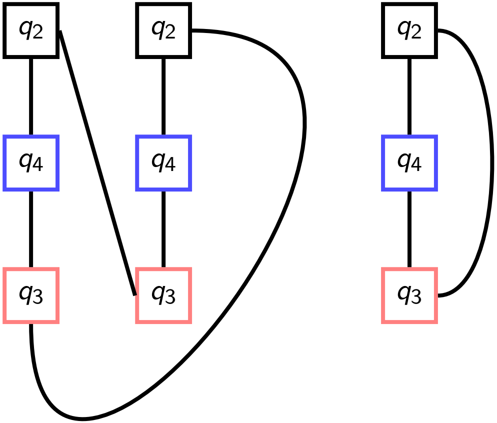

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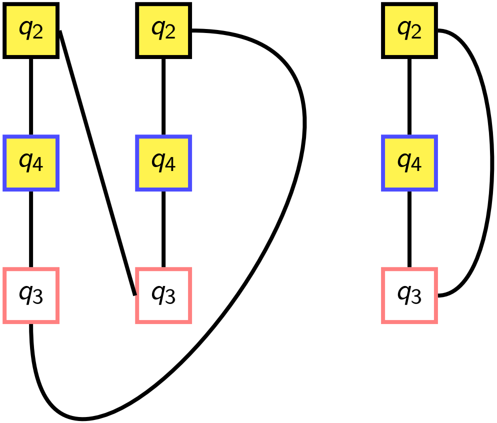

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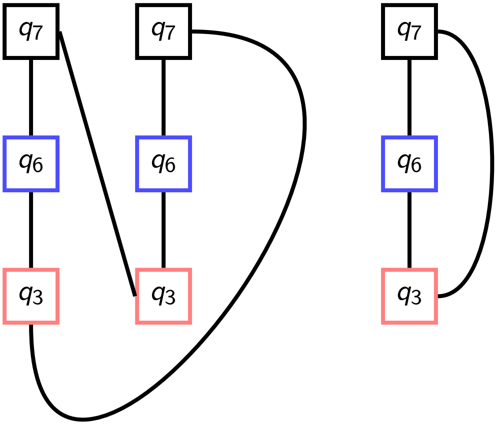

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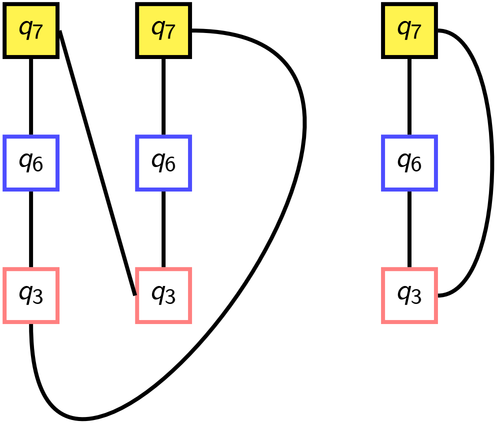

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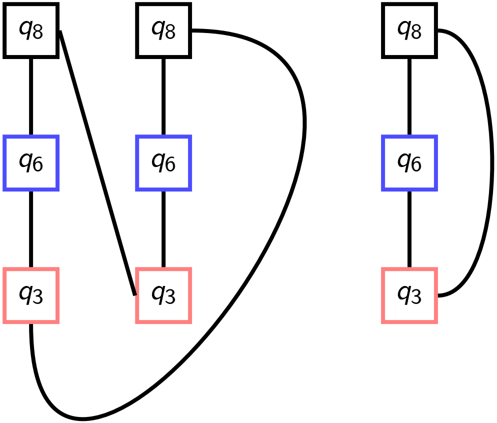

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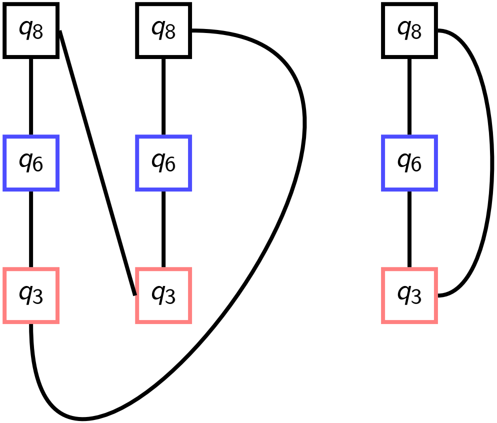
Not if same color nodes are always selected at same time. $\rightarrow$ *Fairness*.



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Detection	Acceptance	Selection	Fairness

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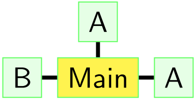
Non-Counting: $\{A, B\}$. 			

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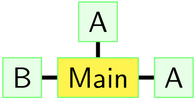
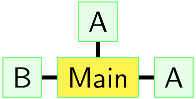
Non-Counting: $\{A, B\}$. 			
Counting: $\{\{A, A, B\}\}$. 			

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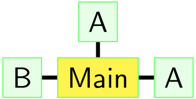
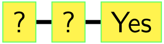
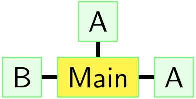
Non-Counting: $\{A, B\}$. 	Halting: Nodes cannot change answer. 		
Counting: $\{\{A, A, B\}\}$. 			

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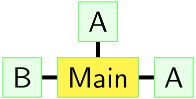
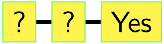
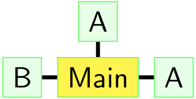
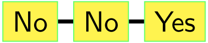
Non-Counting: $\{A, B\}$. 	Halting: Nodes cannot change answer. 		
Counting: $\{\{A, A, B\}\}$. 	Stable Consensus: Nodes can change their answer. 		

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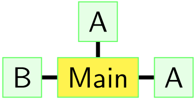
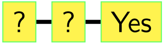
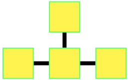
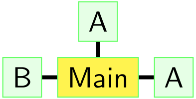
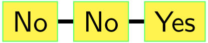
Non-Counting: $\{A, B\}$. 	Halting: Nodes cannot change answer. 	Synchronous: 	
Counting: $\{\{A, A, B\}\}$. 	Stable Consensus: Nodes can change their answer. 		

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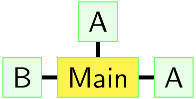
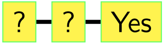
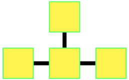
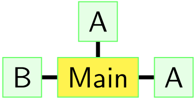
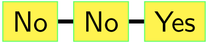
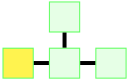
Non-Counting: $\{A, B\}$. 	Halting: Nodes cannot change answer. 	Synchronous: 	
Counting: $\{\{A, A, B\}\}$. 	Stable Consensus: Nodes can change their answer. 	Exclusive: 	

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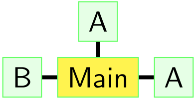
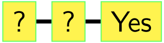
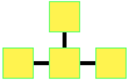
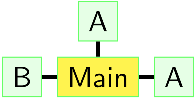
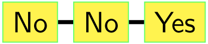
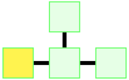
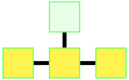
Non-Counting: $\{A, B\}$. 	Halting: Nodes cannot change answer. 	Synchronous: 	
Counting: $\{\{A, A, B\}\}$. 	Stable Consensus: Nodes can change their answer. 	Exclusive: 	
		Liberal: 	

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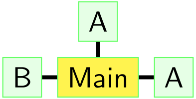
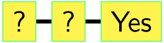
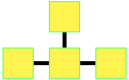
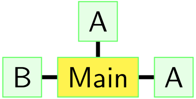
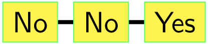
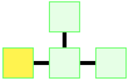
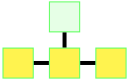
Non-Counting: $\{A, B\}$. 	Halting: Nodes cannot change answer. 	Synchronous: 	Adversarial Scheduling: Every node v is selected infinitely often.
Counting: $\{\{A, A, B\}\}$. 	Stable Consensus: Nodes can change their answer. 	Exclusive: 	
		Liberal: 	

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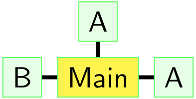
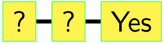
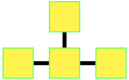
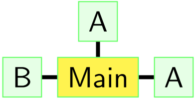
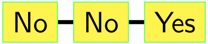
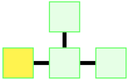
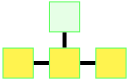
Non-Counting: $\{A, B\}$. 	Halting: Nodes cannot change answer. 	Synchronous: 	Adversarial Scheduling: Every node v is selected infinitely often.
Counting: $\{\{A, A, B\}\}$. 	Stable Consensus: Nodes can change their answer. 	Exclusive: 	Pseudo-Stochastic: Every finite sequence of selections occurs infinitely often.
		Liberal: 	

Classification

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`d`: non-counting	`a`: halting	`f`: adversarial scheduling
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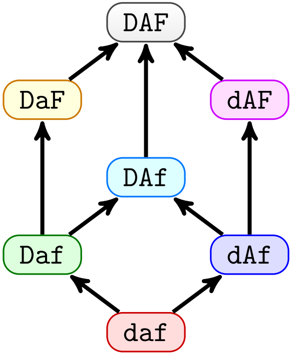

Labeling Properties

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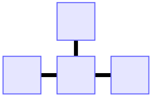
Accepted	

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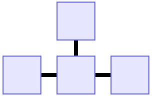
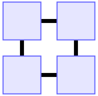
Accepted	
Rejected	

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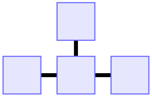
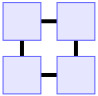
Accepted	
Rejected	

Example	There exists a blue node.

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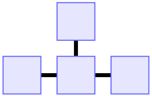
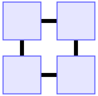
Accepted	
Rejected	

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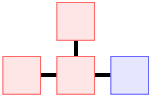
Accepted	

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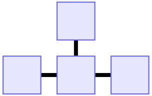
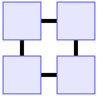
Accepted	
Rejected	

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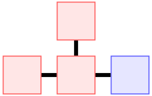
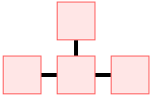
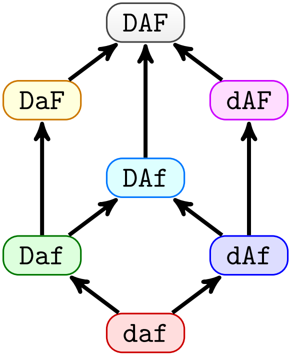
Accepted	
Rejected	

Our Results/Unrestricted Set of Graphs



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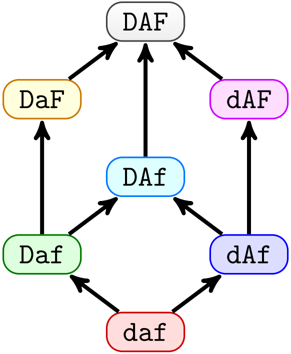
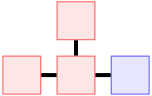
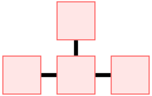


Property	There exists one blue node.
Accepted	
Rejected	

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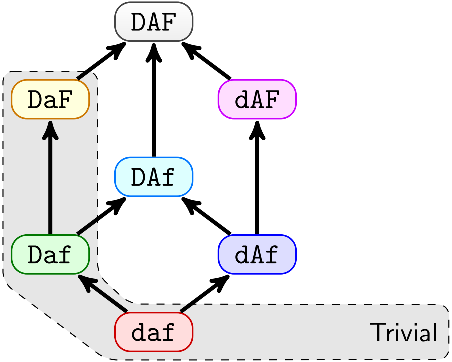
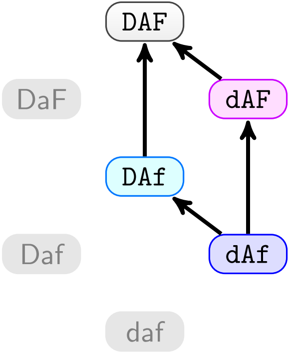
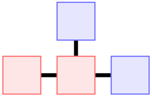
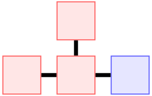


Class	Trivial
includes	True, False

Our Results/Unrestricted Set of Graphs



Property	There exist two blue nodes.
Accepted	
Rejected	

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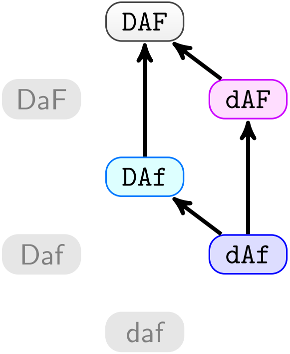

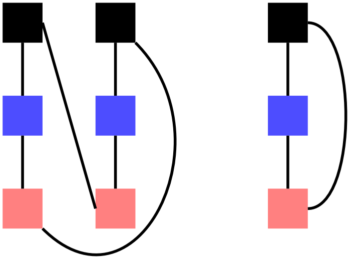

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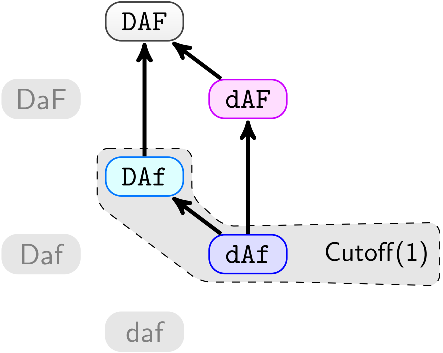
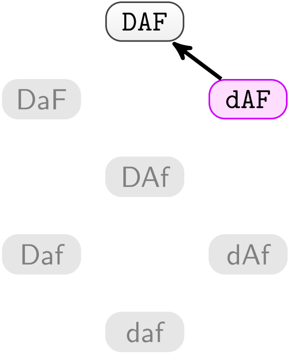
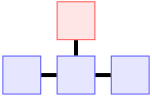
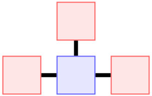


Class	Cutoff(1)
included	There exists both one red and one blue node.
not included	There exist two blue nodes.

Our Results/Unrestricted Set of Graphs



Property	There exist more blue nodes than red nodes.
Accepted	
Rejected	

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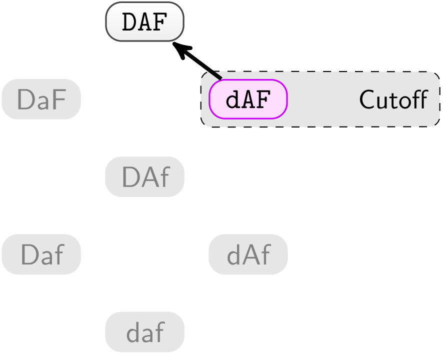
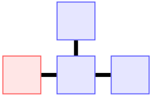
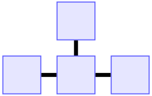


Class	Cutoff
included	There exist three blue nodes and there exist two red nodes.
not included	There exist more blue nodes than red nodes.

Our Results/Unrestricted Set of Graphs

- DAF
- DaF
- dAF
- DAf
- Daf
- dAf
- daf

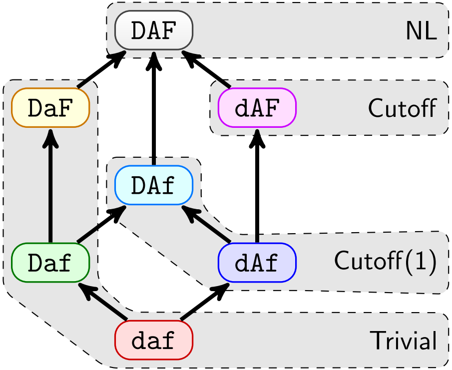
Where is the Limit? What about PRIMES?

Property	The number of blue nodes is a prime number.
Accepted	
Rejected	

Our Results/Unrestricted Set of Graphs

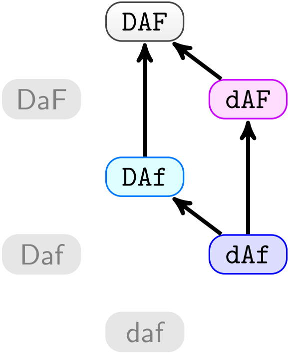
Class	NL = NSPACE($\log n$)
inputs	n blue nodes means input size $\Theta(n)$, i.e. input in unary!
includes	There exist more blue nodes than red nodes. The number of blue nodes is a prime number.

Our Results/Unrestricted Set of Graphs



Class	Distinguishing Property
Cutoff(1)	There exists one blue node.
Cutoff	There exist two blue nodes.
NL	There exist more blue nodes than red nodes.

Our Results/k-Degree-Bounded Graphs



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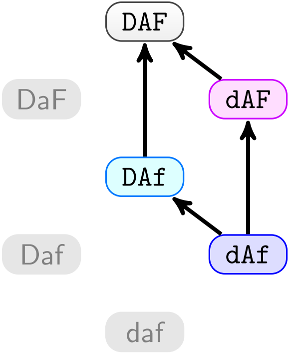

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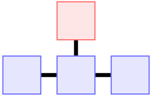
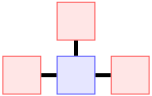
Accepted	
Rejected	

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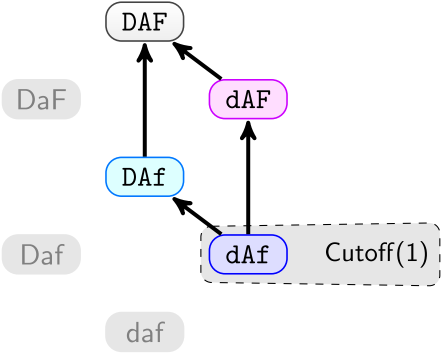
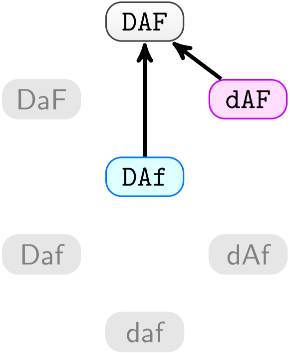


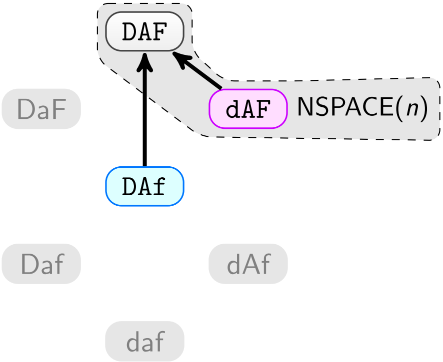
Class	Cutoff(1)
included	There exists both one red and one blue node.
not included	There are two blue nodes. There are more blue nodes than red nodes.

Our Results/k-Degree-Bounded Graphs



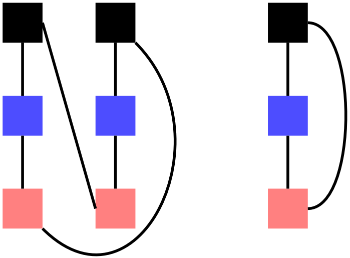
What is the new limit for the strongest model?
Has to be at least NL.

Our Results/k-Degree-Bounded Graphs



Class	NSPACE(n)
How huge?	gigantic. theoretical maximum.

Our Results/k-Degree-Bounded Graphs



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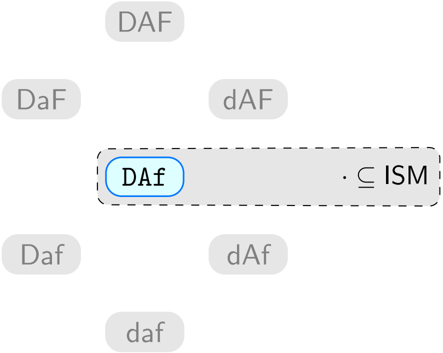
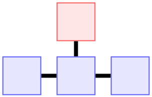
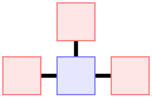


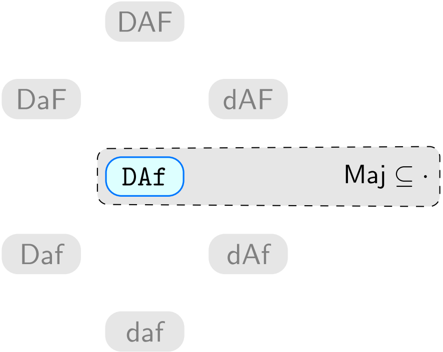
Class	Invariant under Scalar Multiplication
Definition	$\varphi \in \text{ISM} \Leftrightarrow \forall \lambda \in \mathbb{N}_{>0} : \varphi(L) = \varphi(\lambda \cdot L)$.
included	There exist more blue nodes than red nodes.
not included	There exist two blue nodes.

Our Results/k-Degree-Bounded Graphs

Priority: Can we decide Majority with `DAf`?

Property	There exist more blue nodes than red nodes.
Accepted	
Rejected	

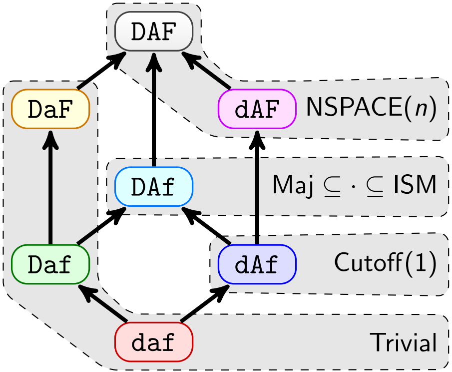
Our Results/k-Degree-Bounded Graphs



Yes!

Can also decide $\sum_i a_i x_i \geq 0$ with $a_i \in \mathbb{Z}$.

Our Results/k-Degree-Bounded Graphs



Class	Distinguishing Property
Cutoff(1)	There exists one blue node.
ISM	There exist more blue nodes than red nodes.
NSPACE(n)	The number of blue nodes is a prime number.

Thank you for your Attention!