Verifying qualitative liveness properties of replicated systems with stochastic scheduling

Martin Helfrich

Joint work with Michael Blondin, Javier Esparza, Antonín Kučera, and Philipp J. Meyer

Technical University of Munich

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- **Stochastic scheduling**: agents to move next are chosen stochastically (every enabled transition has nonzero probability).
- Can model (abstractions of) multithreaded programs, population protocols and other distributed consensus algorithms

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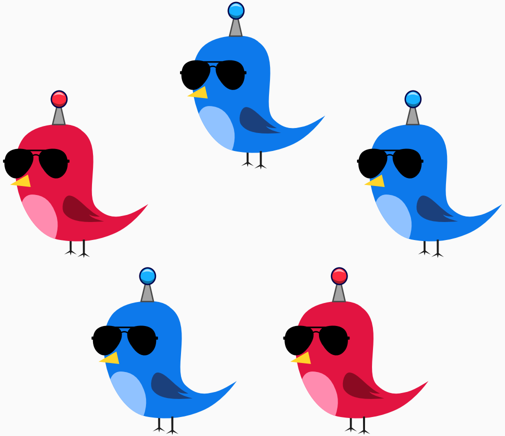
Typical properties: Leader election, consensus

Example: majority voting protocol

At least as many blue birds as red birds?



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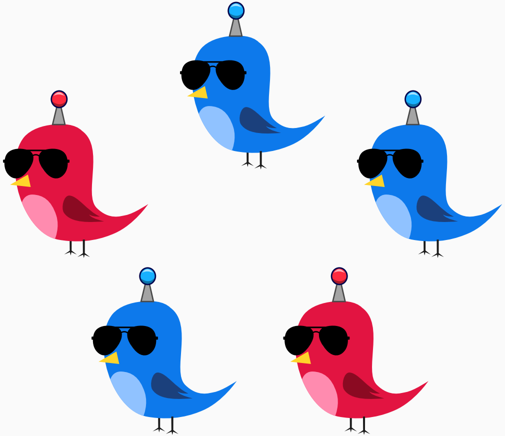

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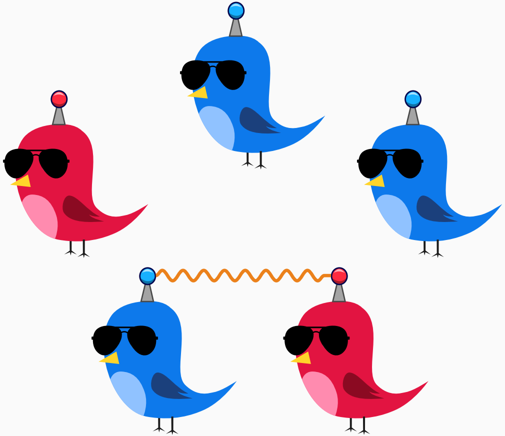

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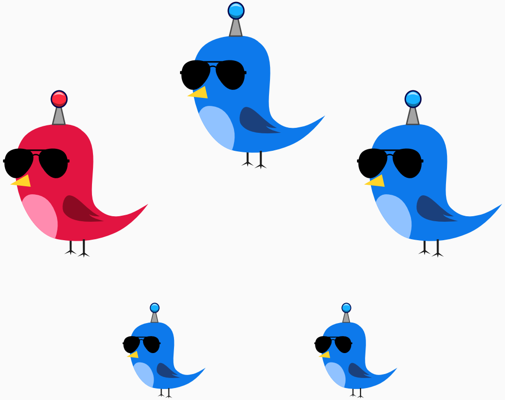

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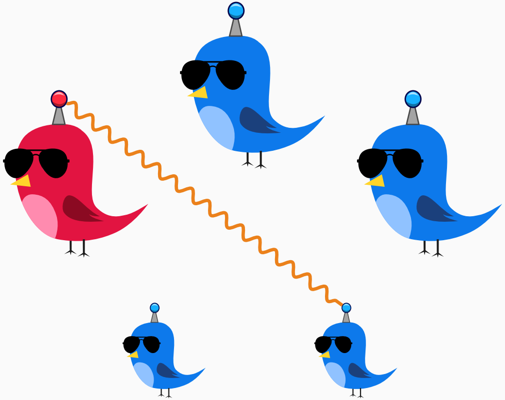

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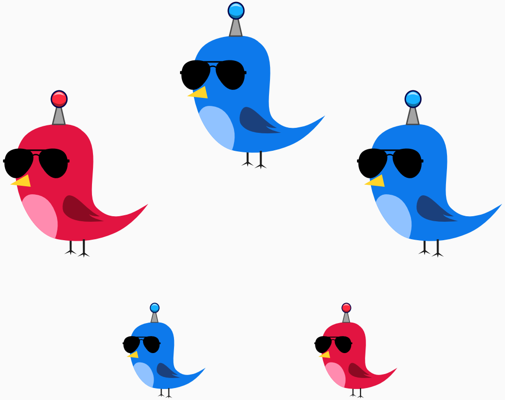

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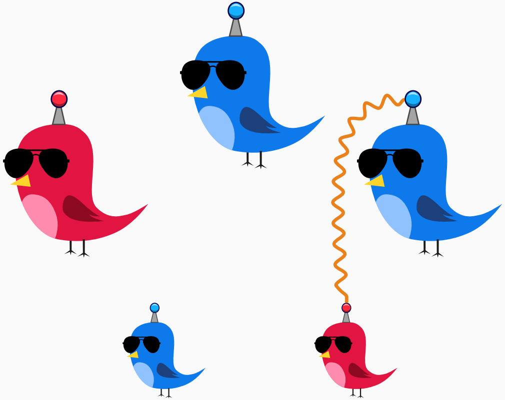

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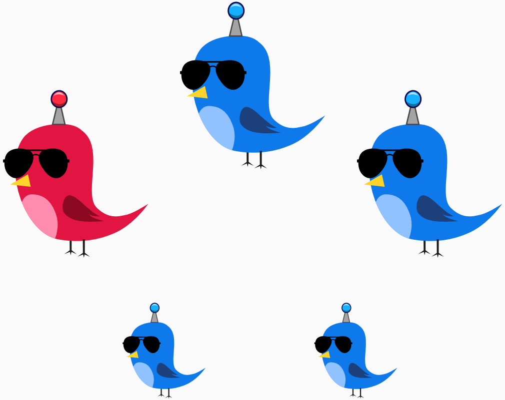

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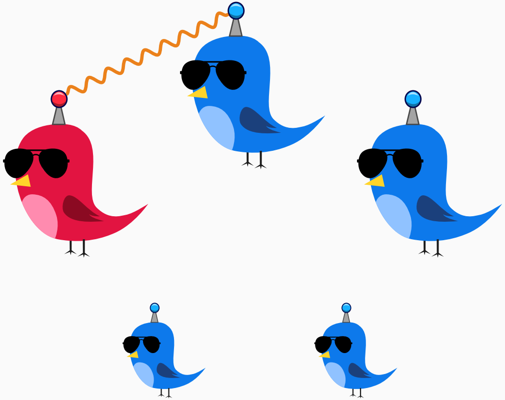

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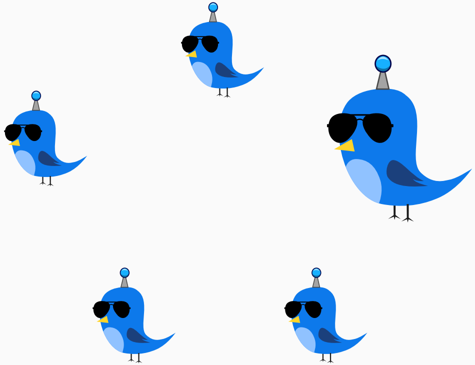

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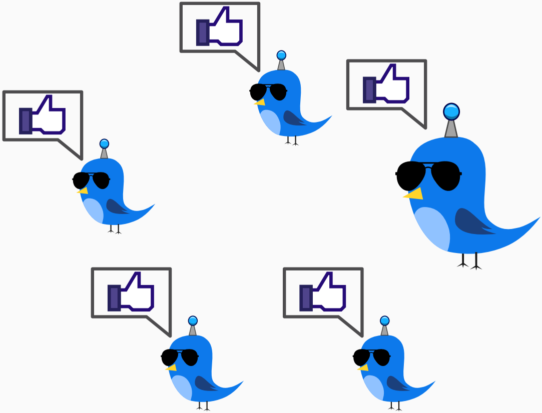

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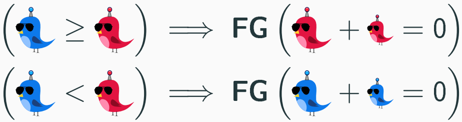
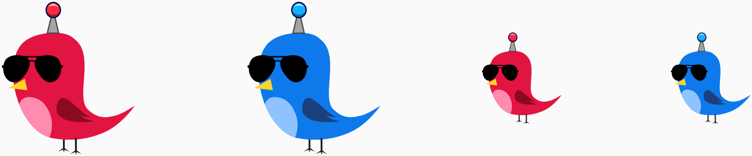
Stable termination properties:



"Birds converge to color of majority."

Replicated Systems: formal model

- States: finite set Q
- Transitions: $T \subseteq \bigcup_{k \geq 2} Q^{\langle k \rangle} \times Q^{\langle k \rangle}$



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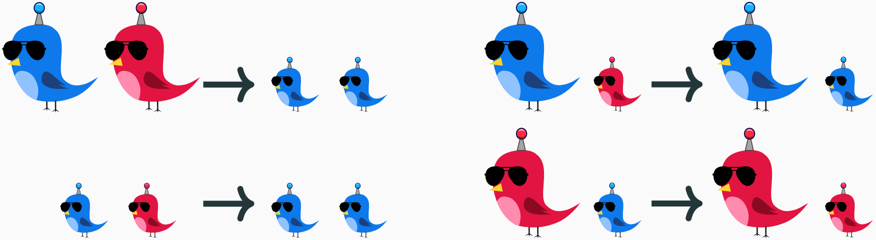

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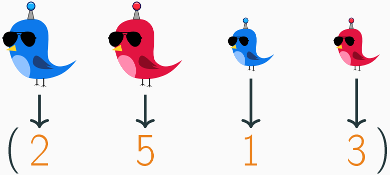

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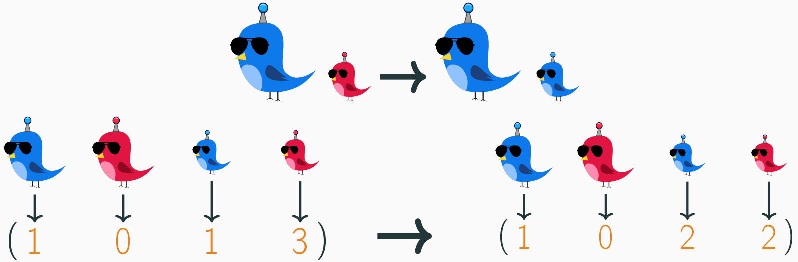

Our Results

We answer two questions:

1. **Theory:** How to verify stable termination?
   $\rightarrow$ sound & complete procedure producing structural proofs

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1. **Theory:** How to verify stable termination?
   $\rightarrow$ sound & complete procedure producing structural proofs
2. **Practice:** How to automatically verify stable termination?
   $\rightarrow$ semi-decision algorithm

Idea

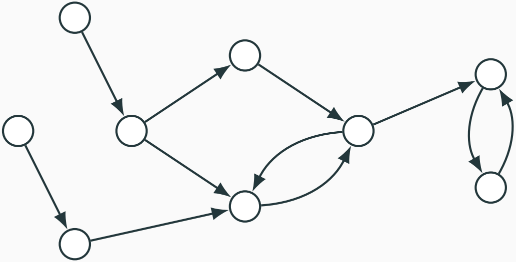
- Most stable termination proofs are structured in stages: milestones trapping the system in increasingly smaller sets of configurations, until it gets trapped in some Post_i

$$\Downarrow$$

Stage Graphs

Preliminaries: inductive set

A (possibly infinite) set of configurations $\mathcal{S}$ is inductive iff it closed under reachability:



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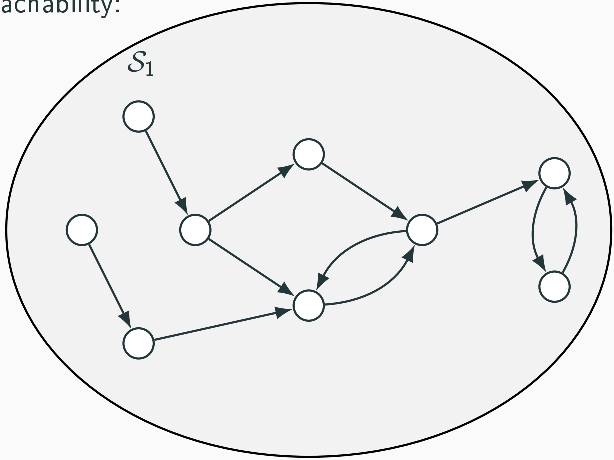

$\mathcal{S}_1$: inductive

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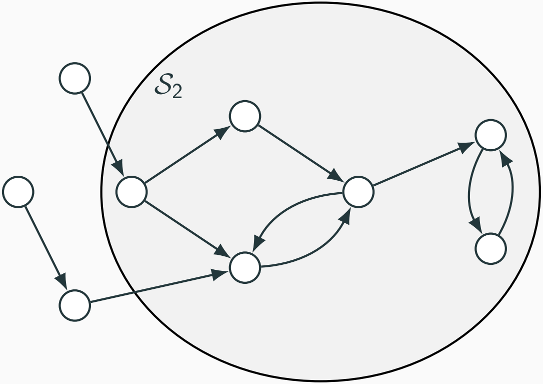

$\mathcal{S}_2$: inductive

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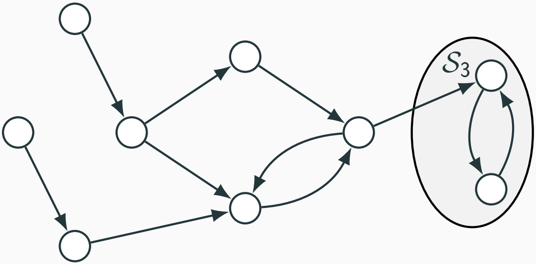

$\mathcal{S}_3$: inductive

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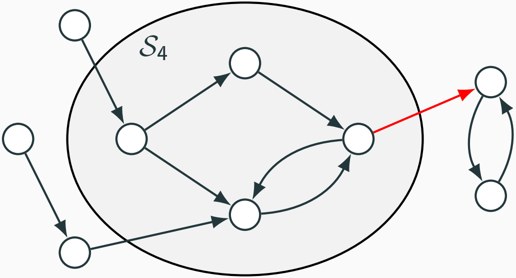

$\mathcal{S}_4$: **not** inductive

Preliminaries: certificate

Let $\mathcal{S}, \mathcal{S}'$ be sets of configurations

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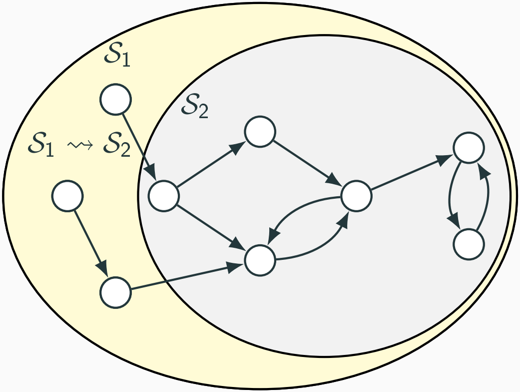

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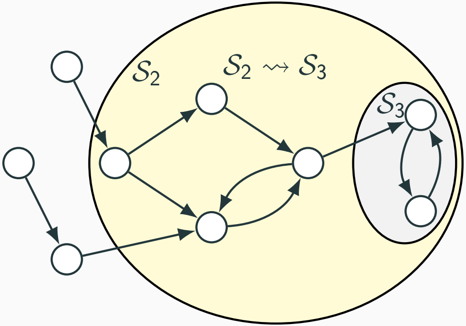

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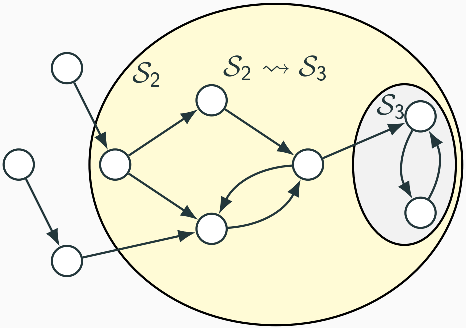

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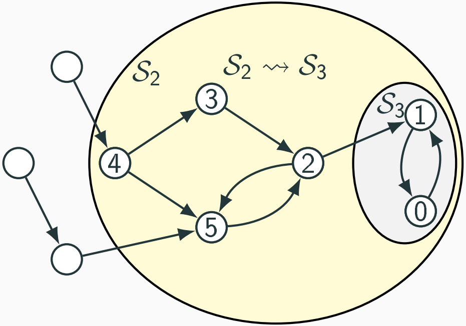

Stage Graphs

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4. Every terminal stage is contained on some Post_i

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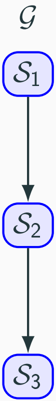

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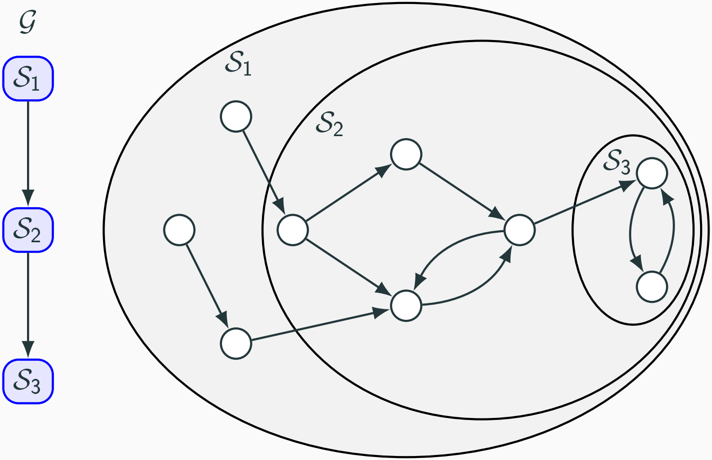

Stages $\mathcal{S}$ of $\mathcal{G}$ are inductive sets

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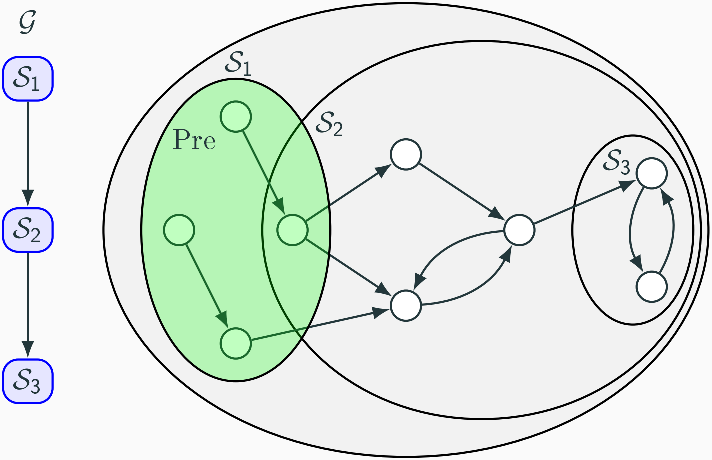

$\mathrm{Pre} \subseteq \bigcup_{\mathcal{S} \in \mathcal{G}} \mathcal{S}$

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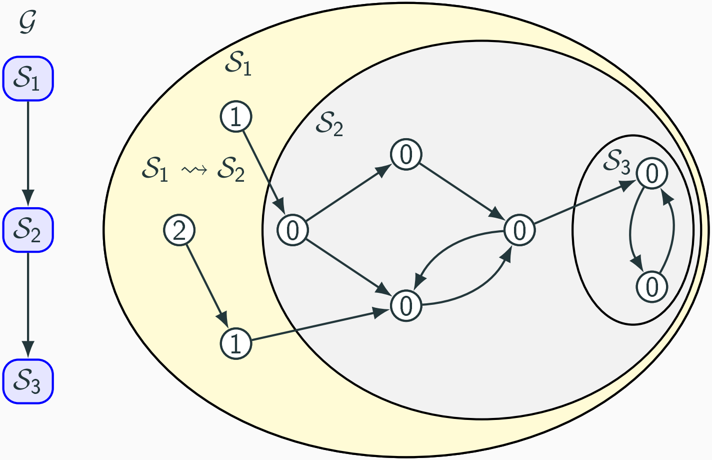

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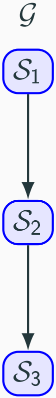

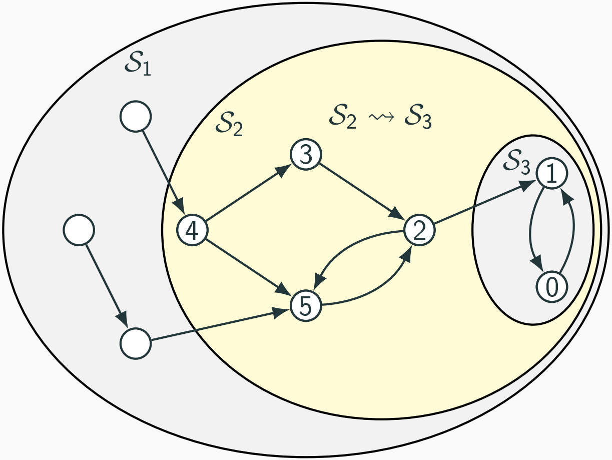

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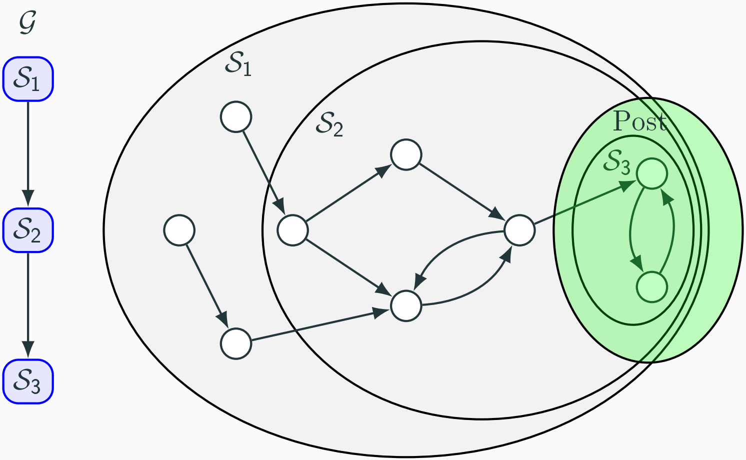
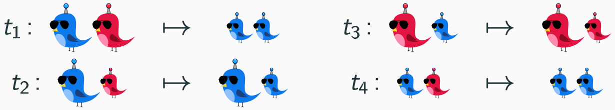


Terminal stages $\mathcal{S} \subseteq \mathrm{Post}$

Stage Graph Example: majority voting protocol



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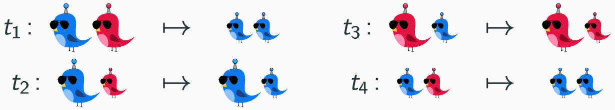

Stage graph for property ( $\geq$ ) $\implies$ **FG** ( $+$  $= 0$)

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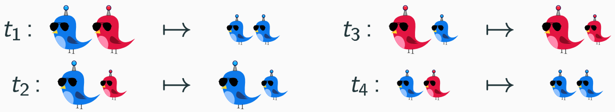

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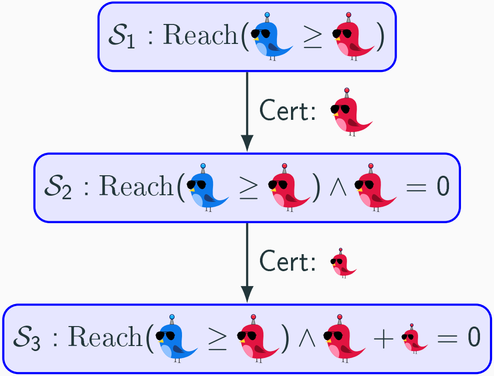

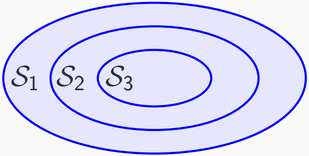

Stage Graphs: theory

Soundness **CAV 2020**

If there is a stage graph for a property, then it holds.

Completeness **CAV 2020**

If a property holds, then there is a stage graph proving it.

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EASY

Completeness **CAV 2020**

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What about decidability?

$\rightarrow$ unknown (stages can be arbitrarily complicated!)

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- nodes are Presburger sets, and

$$C \in \mathcal{S} \iff \phi(C)$$

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HARD

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Presburger stage graph can be independently checked!

$\rightarrow$ everything reduces to checking Presburger formulas

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Decidability **CAV 2020**

It is decidable if a system satisfies a given stable termination property.

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It is decidable if a system satisfies a given stable termination property.

(Alternative) proof.

Two semi-decision algorithms:

- **For non-correctness:** enumerate all configurations and check property (finite-state model checking)
- **For correctness:** enumerate all Presburger stage graphs and check if they prove the property

□

Stage Graphs: theory

Soundness **CAV 2020**

If there is a Presburger stage graph for a property, then it holds.

Completeness **CAV 2020**

If a property holds, then there is a Presburger stage graph proving it.

Decidability **CAV 2020**

It is decidable if a system satisfies a given stable termination property.

Problem: stage graphs might be huge (non-elementary)

$\rightarrow$ How can stage graphs help with automatic verification?

Stage Graphs: practice

Ideas:

- Most systems have small stage graphs

Stage Graphs: practice

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- Most systems have small stage graphs
- Most systems "make progress" by "killing" transitions

Definition

A transition is dead if it can never be enabled again.

Stage Graphs: practice

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- Most systems "make progress" by "killing" transitions
  $\rightarrow$ search for stages with more and more dead transitions

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Stage Graphs: practice

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Algorithm:

SMT based semi-algorithm to automatically construct Presburger stage graphs

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- reachability is TOWER-hard & not Presburger
  $\rightarrow$ overapproximate

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A transition is dead if it can never be enabled again.

$$\Downarrow$$

Algorithm:

SMT based semi-algorithm to automatically construct Presburger stage graphs

- reachability is TOWER-hard & not Presburger
  $\rightarrow$ overapproximate
- use heuristics to find eventually dead transitions
  $\rightarrow$ find linear ranking functions

Experimental Results [1]

Automatically verifying population protocols:

Population Protocol	Predicate	$\|Q\|$	$\|T\|$	Time
Broadcast [31,22]	$x_1 \vee \ldots \vee x_n$	2	1	$< 1s$
Majority (Example 1)[22]	$x \geq y$	4	4	$< 1s$
Majority [23, Ex. 3]	$x \geq y$	5	6	$< 1s$
Majority [5] (m=21,d=20)	$x \geq y$	62	1953	$3301s$
Flock-of-birds [28,22]	$x \geq 80$	81	3240	$1217s$
Flock-of-birds [20, Sect. 3]	$x \geq 120$	9	21	$2551s$
F.o.B. [31,22, threshold-n]	$x \geq 20$	21	39	$18s$
Threshold [8][22]	$\Sigma_i \alpha_i x_i \geq 8$	76	2148	$1089s$
Threshold [20] ("succinct")	$\Sigma_i \alpha_i x_i \geq 511$	25	91	$2659s$
Remainder [22]	$\Sigma_i \alpha_i x_i \equiv_{20} 1$	22	230	$1646s$

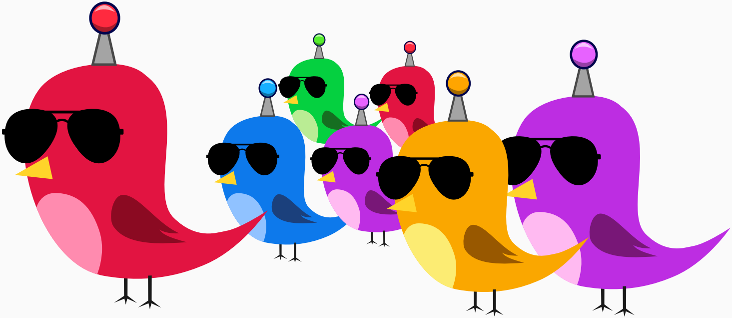
[1] Intel Xeon CPU E5-2630 v4 @ 2.20GHz and 8GB of RAM

Experimental Results [1]

Automatically verifying leader election algorithms:

Algorithm	Processes	$\|Q\|$	$\|T\|$	Time
Israeli-Jalfon [44]	20	40	80	$7s$
Israeli-Jalfon [44]	60	120	240	$1493s$
Israeli-Jalfon [44]	70	140	280	$3295s$
Herman [42]	21	42	42	$9s$
Herman [42]	51	102	102	$300s$
Herman [42]	81	162	162	$2800s$

[1] Intel Xeon CPU E5-2630 v4 @ 2.20GHz and 8GB of RAM



THANK YOU!